

5. Gyakorlat

Diszkrét együttes eloszlás, függetlenség, bevezetés a folytonos eloszlásokhoz
Végeredmények

1. a) nem b)

$Y \backslash X$	0	1	2	p_Y
0	$\frac{9}{36}$	0	0	$\frac{9}{36}$
1	$\frac{12}{36}$	$\frac{6}{36}$	0	$\frac{18}{36}$
2	$\frac{4}{36}$	$\frac{4}{36}$	$\frac{1}{36}$	$\frac{9}{36}$
p_X	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$	(1)

2. a)
- $\frac{1}{60}$
- b)
- $\frac{1}{3}$
- c) igen d)
- $\frac{1}{3}$

3. nem

$Y \backslash X$	1	2	3	4	5
0	$\frac{1}{3}$	$\frac{2}{15}$	$\frac{1}{30}$	0	0
1	0	$\frac{2}{15}$	$\frac{2}{15}$	$\frac{1}{15}$	0
2	0	0	$\frac{1}{30}$	$\frac{1}{15}$	$\frac{1}{15}$

4. a) 2,25 b)
- $\frac{2}{9}$
- c*) 0,5

$Y \backslash X$	0	1	2
0	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{5}$
2	$\frac{1}{20}$	$\frac{1}{4}$	$\frac{1}{5}$

- 5.

nem függetlenek, $\mathbb{P}(XY > 0 \mid X < 2) = \frac{5}{12}$, $\mathbb{E}(4X + Y) = 6$, $\mathbb{E}(XY) = \frac{13}{10}$

6. a) igen b) igen c) nem d) igen

$$7. \quad (a) \quad \alpha = \frac{3}{4}, \quad F(x) = \begin{cases} 0 & x \leq 0 \\ \frac{3}{4}x^2 - \frac{1}{4}x^3 & 0 < x \leq 2 \\ 1 & 2 < x \end{cases}$$

$$(b) \quad \alpha = \frac{3}{2}, \quad F(x) = \begin{cases} 0 & x \leq 2 \\ \sqrt{(x-2)^3} & 2 < x \leq 3 \\ 1 & 3 < x \end{cases}$$

$$(c) \quad \alpha = \frac{3}{2(2\sqrt{2}-1)}, \quad F(x) = \begin{cases} 0 & x \leq 3 \\ \frac{\sqrt{(x-2)^3}-1}{2\sqrt{2}-1} & 3 < x \leq 4 \\ 1 & 4 < x \end{cases}$$

$$(d) \quad \alpha = \frac{1}{2}, \quad F(x) = \begin{cases} 0 & x \leq 0 \\ \sin\left(\frac{x}{2}\right) & 0 < x \leq \pi \\ 1 & \pi < x \end{cases}$$

$$(*e) \quad a) 1 \quad b) 2,63 \quad c) 3,54 \quad d) \frac{\pi}{3}$$

8. 0,1991, nem

9. , nem

10. a) 10,89 b) 3,749

11. (a) $X(\omega) = |\omega - 0.3| = \begin{cases} 0.3 - \omega, & \text{ha } 0 \leq \omega \leq 0.3, \\ \omega - 0.3, & \text{ha } 0.3 \leq \omega \leq 1. \end{cases} \quad (\omega \in \Omega)$

(b) $F_X(x) = \begin{cases} 0, & \text{ha } x \leq 0, \\ 2x, & \text{ha } 0 < x \leq 0.3 \\ 0.6 + (x - 0.3), & \text{ha } 0.3 < x \leq 0.7, \\ 1, & \text{ha } 0.7 < x \end{cases}$

(c) $f_X(x) = \begin{cases} 2, & \text{ha } 0 < x \leq 0.3, \\ 1, & \text{ha } 0.3 < x < 0.7, \\ 0, & \text{egyébként.} \end{cases}$

12. (a) $X(\omega) = \min\{\omega^2, (1 - \omega)^2\} = \begin{cases} \omega^2, & \text{ha } 0 \leq \omega \leq \frac{1}{2}, \\ (1 - \omega)^2, & \text{ha } \frac{1}{2} \leq \omega \leq 1 \end{cases} \quad (\omega \in \Omega)$

(b) $F_X(x) = \begin{cases} 0, & \text{ha } x \leq 0, \\ 2\sqrt{x}, & \text{ha } 0 < x \leq \frac{1}{4}, \\ 1, & \text{ha } \frac{1}{4} < x. \end{cases}$

$f_X(x) = \begin{cases} \frac{1}{\sqrt{x}}, & \text{ha } 0 < x < \frac{1}{4}, \\ 0, & \text{egyébként.} \end{cases}$

13. a) $F_X(x) = \begin{cases} 0, & \text{ha } x \leq 0, \\ 2x - x^2, & \text{ha } 0 < x \leq 1, \\ 1, & \text{ha } x > 1, \end{cases} \quad f_X(x) = \begin{cases} 2 - 2x, & \text{ha } 0 < x < 1, \\ 0, & \text{egyébként.} \end{cases}$

b) $F_Y(y) = \begin{cases} 0, & \text{ha } y \leq 0, \\ y^2, & \text{ha } 0 < y \leq 1, \\ 1, & \text{ha } y > 1, \end{cases} \quad f_Y(y) = \begin{cases} 2y, & \text{ha } 0 < y < 1, \\ 0, & \text{egyébként.} \end{cases}$

14. a) $X(x, y) = y \quad ((x, y) \in \Omega)$

b) $F_X(x) = \begin{cases} 0 & x \leq 0 \\ 4x - 4x^2 & 0 < x \leq \frac{1}{2} \\ 1 & \frac{1}{2} < x \end{cases}, \quad f_X(x) = \begin{cases} 4 - 8x & 0 < x < \frac{1}{2} \\ 0 & \text{egyébként.} \end{cases}$

15. (a) $F_X(x) = \begin{cases} 0 & x \leq 1 \\ 1 + 2\sqrt{x-1} - x & 1 < x \leq 2 \\ 1 & 2 < x \end{cases}$

(b) $f_X(x) = \begin{cases} \frac{1}{\sqrt{x-1}} - 1 & 1 < x < 2 \\ 0 & \text{egyébként} \end{cases}$

(c) 1 körül

16. $F_Y(x) = \begin{cases} 0 & x \leq 0 \\ 3x^2 - 2x^3 & 0 < x \leq 1 \\ 1 & 1 < x \end{cases}, \quad f_Y(x) = \begin{cases} 6x - 6x^2 & 0 < x < 1 \\ 0 & \text{egyébként} \end{cases}$