8. Übungsblatt

Stetige Zufallsvektoren, Randverteilungen, Unabhängigkeit, Faltungsformeln

Ergebnisse

1. a) $\frac{1}{2}$ b) 0 c) 0,4323
2. $\frac{2}{3}$
3. a) $\frac{1}{5}$ b) $\frac{11}{18}$ c) $f_X(s) = \begin{cases} 2s^3 + \frac{1}{2} & \text{falls } 0 < s < 1 \\ 0 & \text{sonst} \end{cases}$ $f_Y(t) = \begin{cases} 2t^3 + \frac{1}{2} & \text{falls } 0 < t < 1 \\ 0 & \text{sonst} \end{cases}$
   d) $\frac{13}{20}$ e) nein f) $\frac{2}{5}$
4. $\mathbb{E}(X) = 2$, $\mathbb{E}(Y) = 1$
5. $\mathbb{P}(X < Y) = \frac{\lambda}{\lambda + \rho}$
6. $\frac{1}{1024}$
7. $a = \frac{1}{5}$, $b = \frac{2}{5}$
8. $Y \sim \text{Exp}(\lambda_1 + \ldots + \lambda_n)$, also $F_Y(t) = 1 - e^{-(\lambda_1 + \ldots + \lambda_n)t}$, $F_Z(t) = \prod_{i=1}^{n}(1 - e^{-\lambda_i t})$

*9.

---

10. $\text{ran}X + Y = \{k \in \mathbb{N}^+ : k \geq 2\}$, $\mathbb{P}(X = k) = (k-1)p^2(1-p)^{k-2}$
11. $\text{Bin}(n + m; p)$
12. $N(0; 1)$
13. $f_{\sqrt{X}}(t) = \begin{cases} \frac{2t^{n-1}e^{-\frac{t^2}{2}}}{2^{\frac{n}{2}}\Gamma\left(\frac{n}{2}\right)}, & \text{fall } t > 0, \\ 0 & \text{sonst} \end{cases}$
14. $f_{X+Y}(t) = \begin{cases} t^2 & \text{falls } 0 < t < 1, \\ 2t - t^2 & \text{falls } 1 < t < 2, \\ 0 & \text{sonst} \end{cases}$
15. a) $f_Z(t) = \begin{cases} t & \text{falls } 0 < t < 1 \\ 2 - t & \text{falls } 1 < t < 2 \\ 0 & \text{sonst} \end{cases}$ $F_Z(t) = \begin{cases} 0 & \text{falls } t < 0 \\ \frac{t^2}{2} & \text{falls } 0 \leq t < 1 \\ -\frac{t^2}{2} + 2t - 1 & \text{falls } 1 \leq t < 2 \\ 1 & \text{falls } 2 \leq t \end{cases}$

    b) $f_Z(t) = \begin{cases} 1 + t & \text{falls } -1 < t < 0 \\ 1 - t & \text{falls } 0 < t < 1 \\ 0 & \text{sonst} \end{cases}$ $F_Z(t) = \begin{cases} 0 & \text{falls } t < -1 \\ t + \frac{t^2}{2} + \frac{1}{2} & \text{falls } -1 \leq t < 0 \\ t - \frac{t^2}{2} + \frac{1}{2} & \text{falls } 0 \leq t < 1 \\ 1 & \text{falls } 1 \leq t \end{cases}$

    c) $f_Z(t) = \begin{cases} \frac{1}{3} + \frac{t}{6} & \text{falls } -2 < t < 0 \\ \frac{1}{3} & \text{falls } 0 < t < 1 \\ \frac{1}{2} - \frac{t}{6} & \text{falls } 1 < t < 3 \\ 0 & \text{sonst} \end{cases}$ $F_Z(t) = \begin{cases} 0 & \text{falls } t < -2 \\ \frac{t^2}{12} + \frac{t}{3} + \frac{1}{3} & \text{falls } -2 \leq t < 0 \\ \frac{t}{3} + \frac{1}{3} & \text{falls } 0 \leq t < 1 \\ -\frac{t^2}{12} + \frac{t}{2} + \frac{1}{4} & \text{falls } 1 \leq t < 3 \\ 1 & \text{falls } 3 \leq t \end{cases}$

16. $f_Z(t) = \begin{cases} \lambda e^{-\lambda t} & \text{falls } 0 < t \\ 0 & \text{sonst} \end{cases}$