Practice session 1

Exercise 1.1: Consider the interpretation $\mathcal{I}_x = \langle \Delta, I \rangle$, where

$$\begin{aligned} \Delta &= \{\texttt{Nick}, \texttt{Mary}, \texttt{Ann}, \texttt{Steve}, \texttt{John}\} \\ \mathsf{Human}^I &= \Delta \\ \mathsf{Female}^I &= \{\texttt{Mary}, \texttt{Ann}\} \\ \mathsf{hasChild}^I &= \{\langle \texttt{Nick}, \texttt{Ann} \rangle, \langle \texttt{Mary}, \texttt{Ann} \rangle, \\ &\quad \langle \texttt{Ann}, \texttt{John} \rangle, \langle \texttt{Steve}, \texttt{John} \rangle\}. \end{aligned}$$

First, evaluate the meanings of the concept expressions appearing on the left and right hand side of the following $\mathcal{AL}$ axioms, in the example interpretation $\mathcal{I}_x$. Next, decide if the interpretation $\mathcal{I}_x$ is a model of the axiom.

(a) $\forall \mathsf{hasChild}.\bot \sqsubseteq \neg \mathsf{Female}$.

(b) $\forall \mathsf{hasChild}.\mathsf{Female} \sqsubseteq \exists \mathsf{hasChild}.\top$.

(c) $\forall \mathsf{hasChild}.\bot \sqsubseteq \forall \mathsf{hasChild}.\mathsf{Female}$.

(d) $\mathsf{Female} \sqcap \forall \mathsf{hasChild}.\mathsf{Female} \sqsubseteq \exists \mathsf{hasChild}.\top$.

(e) $\forall \mathsf{hasChild}.\mathsf{Female} \sqcap \forall \mathsf{hasChild}.\neg \mathsf{Female} \equiv \forall \mathsf{hasChild}.\bot$.

Decide which of the above axioms will be satisfied by *all* their interpretations.

Exercise 1.2: Is there an interpretation which is a model of axiom (b) listed in the above Exercise 1.1? If so, give an example. Try to provide a simpler reformulation of axiom (b).

Exercise 1.3: Consider the DL axioms listed under (a)–(e) in Exercise 1.1, and let $T = \mathsf{Female} \sqsubseteq \exists \mathsf{hasChild}.\top$.

(a) Decide which of the axioms (a)–(e) is a consequence of an empty TBox.

(b) Decide which of the axioms is a consequence of a TBox containing the only axiom T.

(c) Is there an axiom among these which is equivalent to T?

Exercise 1.4: Decide if the following concepts can be formalised using the language $\mathcal{AL}$ and its extensions $\mathcal{C}, \mathcal{U}$, and $\mathcal{E}$. If so, formulate a concept expression corresponding to the given concept.

(a) People with at least one tall child.

(b) People with at most one tall child.

(c) People with at least one child who is either blonde or tall.

(d) People all of whose children are either blonde or tall.

(e) People who do not have at least one tall child.

(f) People none of whose children are either blonde or tall.

Try to give multiple equivalent formalisations of the concepts above. For each, list the single-letter abbreviations of the language extensions it uses.

Exercise 1.5: Using the semantics of $\mathcal{ALC}$ decide which of the following statements hold.

(a) $\{C \equiv \top\} \models \forall R.C \equiv \top$.

(b) $\{\forall R.C \equiv \top\} \models C \equiv \top$.

(c) $\{C \equiv \bot\} \models \exists R.C \equiv \bot$.

(d) $\{\exists R.C \equiv \bot\} \models C \equiv \bot$.

(e) $\models \forall R.C_1 \sqcap \forall R.C_2 \sqsubseteq \forall R.(C_1 \sqcap C_2)$.

(f) $\models \forall R.C_1 \sqcap \forall R.C_2 \sqsupseteq \forall R.(C_1 \sqcap C_2)$.

(g) $\models \exists R.C_1 \sqcap \exists R.C_2 \sqsubseteq \exists R.(C_1 \sqcap C_2)$.

(h) $\models \exists R.C_1 \sqcap \exists R.C_2 \sqsupseteq \exists R.(C_1 \sqcap C_2)$.

(i) $\models \forall R.C_1 \sqcup \forall R.C_2 \sqsubseteq \forall R.(C_1 \sqcup C_2)$.

(j) $\models \forall R.C_1 \sqcup \forall R.C_2 \sqsupseteq \forall R.(C_1 \sqcup C_2)$.

(k) $\models \exists R.C_1 \sqcup \exists R.C_2 \sqsubseteq \exists R.(C_1 \sqcup C_2)$.

(l) $\models \exists R.C_1 \sqcup \exists R.C_2 \sqsupseteq \exists R.(C_1 \sqcup C_2)$.

Exercise 1.6: Decide if the following concepts and statements can be formalised in the $\mathcal{ALCN}$ language. If so, formulate the appropriate concept expression or terminological axiom.

(a) People with at least one blonde child.

(b) People with at most one child.

(c) People with exactly one child.

(d) People with exactly one blonde child.

(e) Mothers having at least three children are optimists.

(f) Anyone can have at most one spouse.

Exercise 1.7: Consider the following roles: hasParent, hasMother, hasFather, hasSpouse, hasAncestor, hasRelative. Build the role hierarchy of this set of roles, i.e. list all the role inclusion axioms involving these roles. Also declare the appropriate roles transitive.

Exercise 1.8: Consider the roles listed in Exercise 1.7, together with the role axioms provided by you as the solution to the exercise. Decide which of the roles are simple.

Exercise 1.9: Formalise the following concepts and statements using the $\mathcal{SHI}$ language. Use the hasParent, hasSibling, and hasPart atomic roles only.

(a) People with at least one tall child.

(b) People who only have tall children.

(c) Children with at least one tall parent.

(d) Parts of a car.

(e) A car which has critical parts that are faulty is itself faulty.

(f) Children of a parent with at least one tall child are either tall themselves or have a tall sibling.

Exercise 1.10: Formalise the following concepts and statements using the $\mathcal{SHIQ}$ language. Use the atomic roles hasParent and hasComponent.

(a) People with at least two tall children.

(b) Cars with at most two faulty components.

(c) A reliable system is not faulty if it has at most one faulty component.[1]

Exercise 1.11: Transform the following English sentences into a TBox using the concepts Human, Man, Woman, MarriedMan, Female, and the role hasWife.

(a) A man is a human who is not female.

(b) A married man is a man.

(c) A married man is someone who has a wife.

(d) A wife of a man is a woman.

(e) A woman is a female human.

(f) No man can have more than one wife.

(g) No woman can be the wife of two men.

Split the above TBox into a definitional part and a background knowledge part. Show that this TBox is definitorial. Make a sample base interpretation and derive the meaning of the name symbols.

Exercise 1.12: Build the expansion of the TBox you supplied as a solution to Exercise 1.11.

[1] Reliable systems use certain techniques, such as duplication of components, to achieve correct behaviour even if some of their components are faulty.

Exercise 1.13: For each of the following reasoning tasks construct a concept C, such that the given reasoning task is equivalent to checking if C is unsatisfiable.

(a) Is $\mathsf{MarriedMan}$ subsumed by Human?

(b) Does $\exists\mathsf{hasWife}.\top$ include Human?

(c) Is $\mathsf{MarriedMan}$ disjoint from Woman?

(d) Are the concepts $\exists\mathsf{hasWife}.\top$ and $\exists\mathsf{hasWife}^{-}.\top$ disjoint?

(e) Are the concepts $\mathsf{Man} \sqcup \mathsf{Woman}$ and Human equivalent?

In the context of the TBox of Exercise 1.11, decide which of the above reasoning tasks should return a positive answer.

Exercise 1.14: Consider the following interpretation $\mathcal{I}_p$:

$$\begin{aligned}
\Delta^{\mathcal{I}_p} &= \{\mathsf{a},\mathsf{b},\mathsf{c},\mathsf{d},\mathsf{e}\};\\
\mathsf{Faulty}^{\mathcal{I}_p} &= \{\mathsf{a},\mathsf{d},\mathsf{e}\};\\
\mathsf{ReliableSystem}^{\mathcal{I}_p} &= \{\mathsf{a}\};\\
\mathsf{hasComponent}^{\mathcal{I}_p} &= \{\langle\mathsf{a},\mathsf{b}\rangle,\langle\mathsf{a},\mathsf{c}\rangle,\langle\mathsf{b},\mathsf{d}\rangle,\langle\mathsf{c},\mathsf{e}\rangle\}.
\end{aligned}$$

Find out the meanings, in interpretation $\mathcal{I}_p$, of the following concept expressions and terminological axioms:

(a) $\mathsf{Faulty} \sqcap \neg\mathsf{ReliableSystem}$.

(b) $\forall\mathsf{hasComponent}^{-}.\neg\mathsf{Faulty}$.

(c) $\exists\mathsf{hasComponent}^{-}.\neg\mathsf{Faulty}$.

(d) $\exists\mathsf{hasComponent}^{-}.\mathsf{Faulty}$.

(e) $(\leqslant 1\mathsf{hasComponent}.\mathsf{Faulty})$.

(f) $(\geqslant 4\mathsf{hasComponent}.\mathsf{Faulty})$.

(g) $(\leqslant 1\mathsf{hasComponent}.\exists\mathsf{hasComponent}.\mathsf{Faulty})$.

(h) $(\geqslant 2\mathsf{hasComponent}.\exists\mathsf{hasComponent}.(\neg\mathsf{Faulty} \sqcup (\geqslant 2\mathsf{hasComponent}^{-}.\top)))$.

(i) $\mathsf{Trans}(\mathsf{hasComponent})$.

(j) $\exists\mathsf{hasComponent}.\mathsf{Faulty} \sqsubseteq \exists\mathsf{hasComponent}^{-}.\mathsf{Faulty}$.

(k) $\exists\mathsf{hasComponent}.\mathsf{Faulty} \sqsubseteq \mathsf{Faulty} \sqcup \exists\mathsf{hasComponent}^{-}.\neg\mathsf{Faulty}$.

(l) $\mathsf{Faulty} \sqcap \mathsf{ReliableSystem} \sqsubseteq (\geqslant 2\mathsf{hasComponent}.\mathsf{Faulty})$.

(m) $\mathsf{ReliableSystem} \sqcup \exists\mathsf{hasComponent}^{-}.\mathsf{ReliableSystem} \equiv \top$.

(n) $\neg\mathsf{Faulty} \sqsubseteq (\leqslant 1\mathsf{hasComponent}.\mathsf{Faulty})$.