Semantic Web – Practice/Homework S1

1. Consider the interpretation $\mathcal{I}_x = \langle \Delta, I \rangle$, where

$$\begin{aligned} \Delta &= \{\texttt{Nick}, \texttt{Mary}, \texttt{Ann}, \texttt{Steve}, \texttt{John}\} \\ \mathsf{Female}^I &= \{\texttt{Mary}, \texttt{Ann}\} \\ \mathsf{hasChild}^I &= \{\langle \texttt{Nick}, \texttt{Ann} \rangle, \langle \texttt{Mary}, \texttt{Ann} \rangle, \\ & \quad \langle \texttt{Ann}, \texttt{John} \rangle, \langle \texttt{Steve}, \texttt{John} \rangle\}. \end{aligned}$$

First, evaluate the meanings of the concept expressions appearing on the left and right hand side of the following $\mathcal{AL}$ axioms, in the example interpretation $\mathcal{I}_x$. Next, decide if the interpretation $\mathcal{I}_x$ is a model of the axiom.

a) $\forall \mathsf{hasChild}.\bot \sqsubseteq \neg \mathsf{Female}$.

b) $\forall \mathsf{hasChild}.\mathsf{Female} \sqsubseteq \exists \mathsf{hasChild}.\top$.

c) $\forall \mathsf{hasChild}.\bot \sqsubseteq \forall \mathsf{hasChild}.\mathsf{Female}$.

d) $\mathsf{Female} \sqcap \forall \mathsf{hasChild}.\mathsf{Female} \sqsubseteq \exists \mathsf{hasChild}.\top$.

e) $\forall \mathsf{hasChild}.\mathsf{Female} \sqcap \forall \mathsf{hasChild}.\neg \mathsf{Female} \equiv \forall \mathsf{hasChild}.\bot$.

Select those of the above axioms which are tautologies, i.e. statements that hold in **all** their interpretations. You may check your results using Protégé.

2. Decide if the following concepts and statements (the last three, below the line) can be formalised in the language $\mathcal{AL}$ and its extensions $\mathcal{C}$, $\mathcal{U}$, $\mathcal{E}$ and $\mathcal{N}$. If so, formulate the appropriate concept expression or axiom. If not, just write: "cannot be formulated".
You can assume that the universe contains people only, so you do not need to mention the the concept Person in the formulas. Try to use the simplest language possible, e.g. use $\mathcal{AL}$ with the smallest number of extensions. For each formula list the single-letter abbreviations of the $\mathcal{AL}$ extensions it uses.

a) People with at least one tall child.
Solution: $\exists\mathsf{hasChild}.\mathsf{Tall}$ – $\mathcal{E}$

b) People who do not have at least one tall child.
Solution: $\neg\exists\mathsf{hasChild}.\mathsf{Tall}$ – $\mathcal{EC}$
Reformulation: $\forall\mathsf{hasChild}.\neg\mathsf{Tall}$ – just plain $\mathcal{AL}$ (without any extensions)

c) People with at most one tall child (may have several children, but at most one of these is tall).
Solution: cannot be formulated in $\mathcal{ALCUEN}$

d) People with at least one child who is either blonde or tall.

e) People each of whose children is either blonde or tall.

f) People none of whose children are either blonde or tall.

g) People with at most one child.

h) People with exactly one child.

i) People with exactly one child who happens to be blonde.

j) People with exactly one blonde child (may have several children, but exactly one of these is blonde).

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k) A mother having at least three children is an optimist.

l) A woman is an optimist if all her children are optimists.
(Note that this is the same as: if all children of a woman are optimists then she herself is an optimist.)

m) Anyone can have at most one spouse.

3. Using the semantics of $\mathcal{ALC}$ decide which of the following axioms are tautologies, i.e. statements that hold in **all** their interpretations.

   (a) $\forall R.C_1 \sqcap \forall R.C_2 \sqsubseteq \forall R.(C_1 \sqcap C_2)$.

   (b) $\forall R.C_1 \sqcap \forall R.C_2 \sqsupseteq \forall R.(C_1 \sqcap C_2)$.

   (c) $\exists R.C_1 \sqcap \exists R.C_2 \sqsubseteq \exists R.(C_1 \sqcap C_2)$.

   (d) $\exists R.C_1 \sqcap \exists R.C_2 \sqsupseteq \exists R.(C_1 \sqcap C_2)$.

   (e) $\forall R.C_1 \sqcup \forall R.C_2 \sqsubseteq \forall R.(C_1 \sqcup C_2)$.

   (f) $\forall R.C_1 \sqcup \forall R.C_2 \sqsupseteq \forall R.(C_1 \sqcup C_2)$.

   (g) $\exists R.C_1 \sqcup \exists R.C_2 \sqsubseteq \exists R.(C_1 \sqcup C_2)$.

   (h) $\exists R.C_1 \sqcup \exists R.C_2 \sqsupseteq \exists R.(C_1 \sqcup C_2)$.

   You may check your results using Protégé.

4. Consider the following roles: hasParent, hasMother, hasChild, hasFather, hasGrandparent, hasAncestor, hasBloodRelation (People x and y are blood relations if they have a common ancestor).
   Build the role hierarchy of this set of roles, i.e. list all the role inclusion axioms involving these roles. Also declare the appropriate roles transitive.

5. Consider the roles listed in Exercise 4, together with the role axioms provided by you as the solution to the exercise. Decide which of the roles are simple.

6. Formalise the following concepts and statements using the $\mathcal{SI}$ language. Use the hasParent, hasSibling, and hasPart atomic roles and the Person, Tall, Car, Critical, Faulty, atomic concepts only.

   a) People with at least one tall child.

   b) People who only have tall children.

   c) Children with at least one tall parent.

   d) Parts of a car.

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   e) A car which has a critical part that is faulty is itself faulty.

   f) Children of a person with at least one tall child are either tall themselves or have a tall sibling.

7. Formalise the following concepts and statements using the $\mathcal{SHIQ}$ language. Use the atomic roles hasParent and hasComponent. The concept names to be used are given at each task.

   a) People with at least two tall children. (Person, Tall)

   b) Cars with at most two faulty components. (Car, Faulty)

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   c) A reliable system is not faulty if it has at most one faulty component. (ReliableSystem, Faulty)
   (Reliable systems use certain techniques, such as duplication of components, to achieve correct behaviour even if some of their components are faulty.)