AIT Semantic and Declarative Technologies Course

Homework L4: Defining family relationships using FOL clauses

Assume the definition of the following predicates has been provided:

- `hParent(Ch, P)`: `Ch` has a parent `P`.
- `female(P)`: `P` is female.
- `male(P)`: `P` is male.

For example these predicates can be defined in "database style" by the following clauses:

```
+hParent(a, f).          +male(a).        +female(c).
+hParent(a, g).          +male(b).        +female(d).
+hParent(b, f).          +male(f).        +female(g).
...                      ...              ...
```

Your task is to write clauses defining predicates for the following family relationships:

1. `hGrandparent(Ch, G)`: `Ch` has a grandparent `G`. (solved below as an example)
2. `hMother(Ch, M)`: `Ch`'s mother is `M`.
3. `hChild(P, Ch)`: `P` has a child `Ch`.
4. `hSon(P, S)`: `P` has a son `S`.
5. `hSibling(S1, S2)`: `S1` has a sibling `S2`.
   (Clarification: let's consider two people siblings, if they share *at least* one parent.)
6. `hCousin(C1, C2)`: `C1` has a cousin `C2`.
   (Clarification: cousins have parents who are siblings, in the above sense.)
7. `hAncestor(Ch, A)`: `Ch` has an ancestor `A`.
   (Clarification: a person's ancestors are: their parent, their parent's parent, their parent's parent's parent, etc.)

Note that each of these definitions, except for the last, should be transformed to a single clause.
Suggested steps, using the first task as an example:

- Start with writing a semi-formalized English sentence which describes how to check the given family relationship in terms of one or more other ones. The sentence should start with the specification of the predicate (the English sentence after the colon), followed by the word **if**:

  `Ch` has a grandparent `G` **if** (...).

  The remaining part, (...), should take the form of one or more sentences separated by **and** (possibly also by **or**). These sentences should be selected from the ones appearing after a colon in the above descriptions of the predicates. What you may change is only the naming of the individuals concerned.

  Also, you may need to add the text: "there exist(s) `X` ... such that" in front of these sentences.

  In the example:

  `Ch` has a grandparent `G` **if** there exists `P` such that `Ch` has a parent `P` **and** `P` has a parent `G`.

- Rewrite the text into a FOL implication $\leftarrow$ (drawn right to left):

  $\forall \texttt{Ch}, \texttt{G}.(\texttt{hGrandparent}(\texttt{Ch}, \texttt{G}) \leftarrow \exists \texttt{P}.(\texttt{hParent}(\texttt{Ch}, \texttt{P}) \wedge \texttt{hParent}(\texttt{P}, \texttt{G})))$.

- Rewrite the above formula to clausal form:
  - Rewrite the implication to a disjunction:

    $\forall \texttt{Ch}, \texttt{G}.(\texttt{hGrandparent}(\texttt{Ch}, \texttt{G}) \vee \neg\exists \texttt{P}.(\texttt{hParent}(\texttt{Ch}, \texttt{P}) \wedge \texttt{hParent}(\texttt{P}, \texttt{G})))$.
  - Move negation inside non-atomic formulas:

    $\forall \texttt{Ch}, \texttt{G}.(\texttt{hGrandparent}(\texttt{Ch}, \texttt{G}) \vee \forall \texttt{P}.(\neg\texttt{hParent}(\texttt{Ch}, \texttt{P}) \vee \neg\texttt{hParent}(\texttt{P}, \texttt{G})))$.
  - Bring quantifiers outside:

    $\forall \texttt{Ch}, \texttt{G}, \texttt{P}.(\texttt{hGrandparent}(\texttt{Ch}, \texttt{G}) \vee \neg\texttt{hParent}(\texttt{Ch}, \texttt{P}) \vee \neg\texttt{hParent}(\texttt{P}, \texttt{G}))$.
  - Transform to clause notation: `+hGrandparent(Ch, G) -hParent(Ch, P) -hParent(P, G).`

  Note that you need not go through the details of this transformation, it is enough to provide the final form in clause notation.

For the predicate `hSibling` you need one more helper predicate:

`neq(X,Y)`: `X` is not equal to `Y`.

You can think of this predicate as if defined by the following FOL formula:

$\texttt{neq}(x, y) \leftarrow \neg(x = y)$