

AIT Semantic and Declarative Technologies Course

Optional Practice L3-2: Propositional Resolution

(3 optional points per puzzle solved)

Inspector Craig puzzles

Below is a set of problems from Raymond Smullyan's "What is the name of this book". The chapter is entitled "From the cases of Inspector Craig". Each puzzle involves some suspects of a crime, named A, B, etc. Some of them are guilty, some innocent. There are some statements given.

Example

An enormous amount of loot had been stolen from a store. The criminal (or criminals) took the heist away in a car. Three well-known criminals A, B, C were brought to Scotland Yard for questioning. The following facts were ascertained:

- (1) No one other than A, B, C was involved in the robbery.
- (2) C never works without A (and possibly others) as an accomplice.
- (3) B does not know how to drive.

Is A innocent or guilty?

Your tasks:

- (i) Try to solve the puzzle as a recreational task :-).
- (ii) Transform each statement into a propositional formula involving the letters A, B, etc. as atomic propositions. Proposition X stands for "X is guilty".
 - a) A is guilty or B is guilty or C is guilty: $A \vee B \vee C$
 - b) If C is guilty then A is guilty: $C \rightarrow A$
 - c) It cannot be the case that only B is guilty: $B \rightarrow (A \vee C)$
- (iii) Transform each propositional formula into conjunctive normal form (CNF), then show the clauses in simplified form:
 - a) $A \vee B \vee C$ (already in CNF), clauses: +A +B +C.
 - b) $C \rightarrow A$, CNF: $\neg C \vee A$, clauses: -C +A.
 - c) $B \rightarrow (A \vee C)$, CNF: $\neg B \vee A \vee C$, clauses: -B +A +C.(Note that in general a single statement can give rise to multiple clauses.)
- (iv) Collect the clauses, give each a reference number and perform a resolution proof:

(1)	+A +B +C.				
(2)	-C +A.				
(3)	-B +A +C.				
(4)	+A +C.	(1)/2 rw (3)/1	read	clause (1) literal 2 resolved with	clause (3) literal 1
(5)	+A.	(4)/2 rw (2)/1			
- (v) Remove the clauses implied by other clauses:

Clauses (1) – (4) each contain the literal +A, so they all are a trivial consequence of (5). Hence clauses (1) – (4) can be removed.
- (vi) Interpret the answer: A is guilty.

Your task:

Perform the above steps for the following six puzzles.

For steps (iii)–(v) we suggest to use the Plwin tool: <https://ait.plwin.dev/L3-2>.

Enter the conjunction of the formulas you obtained in step (ii) into the tool and then perform the steps (iii)–(v) within the tool. If you do so, there is no need to submit any further material.

1. Another robbery. A, B, and C were brought in for questioning and the following facts were found:

- (1) No one other than A, B, and C was involved.
- (2) A never works without at least one accomplice.
- (3) C is innocent.

Is B innocent or guilty?

2. "What do you make of these three facts?" asked Inspector Craig to Sergeant McPherson.

- (1) If A is guilty and B is innocent, then C is guilty.
- (2) C never works alone.
- (3) A never works with C.
- (4) No one other than A, B, or C was involved, and at least one of them is guilty.

The Sergeant scratched his head and said, "Not much, I'm afraid. Can you tell from these facts which are innocent and which are guilty?" "No," said Craig, "but there is enough to know that one of them is guilty." Which is it?

3. This time four suspects A, B, C, D were rounded up for questioning after a robbery. It is known for sure that at least one of them is guilty and that no one outside the four is involved. The following facts appeared:

- (1) A was definitely innocent.
- (2) If B was guilty, then he had exactly one accomplice.
- (3) If C was guilty, then he had exactly two accomplices.

Inspector Craig especially wanted to know whether D was guilty, because he was a particularly dangerous criminal. Is D guilty or not?

4. Three men A, B, and C are on trial for participation in a robbery. In this case, the following two facts were established:

- (1) If A is innocent or B is guilty, then C is guilty.
- (2) If A is innocent, then C is innocent.

Can the guilt of any one of the three be established?

5. In this case, there are four defendants, A, B, C, and D. The following facts are established:

- (1) If A is guilty, then B was an accomplice.
- (2) If B is guilty then either C was an accomplice or A is innocent.
- (3) If D is innocent then A is guilty and C is innocent.
- (4) If D is guilty, so is A.

Which ones are innocent and which are guilty?

6. Mr. McGregor, a London shopkeeper, phoned the police and said that his shop had been robbed. Three suspects A, B, and C were rounded up for questioning. The following facts were established:

- (1) Each of the men A, B, C had been in the shop on the day of the robbery, and no one else had been there on that day.
- (2) If A was guilty, then he had exactly one accomplice.
- (3) If B is innocent, so is C.
- (4) If exactly two are guilty, then A is one of them.
- (5) If C is innocent, so is B.

Whom did Inspector Craig indict? :-)