AIT Semantic and Declarative Technologies Course
Practice L2: Propositional Logic – expressing Boolean functions

1. Implication (Recall that $U \to V = \neg U \vee V$.)

   a) Express implication using negation and conjunction only:
   $(U \to V) =$

   b) Express disjunction using implication and negation only:
   $(U \vee V) =$

2. Equivalence

   a) Express equivalence using implication and conjunction:
   $(U \leftrightarrow V) =$

   b) Express equivalence using negation, disjunction and conjunction:
   $(U \leftrightarrow V) =$

3. Exclusive Or

   a) Express "exclusive or" ($\oplus$) using negation and equivalence only:
   $(U \oplus V) =$

   b) Express "exclusive or" using negation, conjunction and disjunction:
   $(U \oplus V) =$

4. Complex Boolean expressions (you can use arbitrary operations introduced above, try to find the simplest formula)

   a) Express the condition that exactly one of the propositions A and B are true

   b) Express the condition that exactly two of the propositions A and B and C are true

   c) Express the boolean if-then-else function, similar to the C language expression (`x ?  y :  z`):

   $$(\text{if } A \text{ then } B \text{ else } C) = \begin{cases} B & \text{if } A \text{ is true} \\ C & \text{otherwise} \end{cases}$$