Semantic and Declarative Technologies

Final exam sample

Time: 80 minutes.

This is an open book test, you can use the printed version of the slides and/or the textbook. You are not allowed to use electronic equipment (notebooks, phones, etc.)

1. Consider the interpretation $\mathcal{I} = \langle \Delta, I \rangle$, where

$$\begin{aligned} \Delta &= \{\texttt{Nick}, \texttt{Mary}, \texttt{Ann}, \texttt{Steve}, \texttt{John}\} \\ \textsf{Female}^I &= \{\texttt{Mary}, \texttt{Ann}\} \\ \textsf{hasChild}^I &= \{\langle \texttt{Nick}, \texttt{Ann} \rangle, \langle \texttt{Mary}, \texttt{Ann} \rangle, \\ &\quad \langle \texttt{Ann}, \texttt{John} \rangle, \langle \texttt{Steve}, \texttt{John} \rangle\} \\ \textsf{Optimist}^I &= \{\texttt{Nick}, \texttt{Steve}, \texttt{Ann}\} \end{aligned}$$

Consider also the TBox $\mathcal{T}$ consisting of the following three axioms:

$$\textsf{Father} \equiv \neg\textsf{Female} \sqcap \exists\textsf{hasChild}.\top \tag{1}$$

$$\textsf{FatherOfGirls} \equiv \textsf{Father} \sqcap \forall\textsf{hasChild}.\textsf{Female} \tag{2}$$

$$\textsf{FatherOfGirls} \sqsubseteq \textsf{Optimist} \sqcup \exists\textsf{hasChild}.\textsf{Optimist}. \tag{3}$$

Statements (1) and (2) are definitional axioms, while (3) is a General Concept Inclusion axiom.

(a) Extend the interpretation $\mathcal{I}$ to include the meaning of the concepts Father and FatherOfGirls so that axioms (1) and (2) hold.

$$\begin{aligned} \textsf{Father}^I &= \{\texttt{Nick}, \texttt{Steve}\} \\ \textsf{FatherOfGirls}^I &= \{\texttt{Nick}\}. \end{aligned}$$

(b) Evaluate the meaning of the concept expression on the right hand side of (3), i.e. provide the subset of Δ that corresponds to this concept expression in $\mathcal{I}$.

$$(\textsf{Optimist} \sqcup \exists\textsf{hasChild}.\textsf{Optimist})^I = \{\texttt{Nick}, \texttt{Steve}, \texttt{Ann}, \texttt{Mary}\}$$

(c) Decide if (3) holds in $\mathcal{I}$.

It holds

2. Build a $\mathcal{SHIQ}$ TBox, representing the statements below. You can only use the concept names shown in grotesque font, and the role names hasChild and hasFriend.

   A Person is Lucky if she or he has a Happy grandparent and has at least two Clever children.

$$\mathsf{Person} \sqcap (\exists \mathsf{hasChild}^{-}.\exists \mathsf{hasChild}^{-}.\mathsf{Happy}) \sqcap (\geqslant 2\,\mathsf{hasChild}.\mathsf{Clever}) \quad \sqsubseteq \quad \mathsf{Lucky}$$

   We know that a Lucky Person either has no friends or all her/his parents are not Happy.

$$\mathsf{Person} \sqcap \mathsf{Lucky} \quad \sqsubseteq \quad (\forall \mathsf{hasFriend}.\bot) \sqcup (\forall \mathsf{hasChild}^{-}.\neg \mathsf{Happy})$$

   We also know that no one can be both Clever and Lucky.

$$\mathsf{Clever} \sqcap \mathsf{Lucky} \quad \sqsubseteq \quad \bot$$

   Friendship is symmetric, i.e. if x has a friend y, then y is bound to have x as a friend.

$$\mathsf{hasFriend}^{-} \quad \sqsubseteq \quad \mathsf{hasFriend}$$

   Parents are always befriended with their children.

$$\mathsf{hasChild} \quad \sqsubseteq \quad \mathsf{hasFriend}$$

3. Consider the following subsumption check task, with respect to an empty TBox:

$$\exists \mathsf{hC}.(\mathsf{B} \sqcap (\geqslant 1\,\mathsf{hF})) \sqcap \exists \mathsf{hC}.(\mathsf{B} \sqcap \forall \mathsf{hF}.\bot) \sqcap (\leqslant 2\mathsf{hC}) \sqsubseteq \forall \mathsf{hC}.\mathsf{B}.$$

   (a) Rewrite the above axiom to a sentence in English (hC stands for "has child", hF stands for "has friend" and B stands for "blonde").
   Use the following format: "If someone ... then ...".
   If someone (x) has a child who is blonde and who has at least 1 friend
   and the same person x also has a child who is blonde and who has no friends
   and the same person x has at most 2 children
   then all children of this person x are blonde

   (b) Is the above statement true? (A "yes" or "no" answer is enough.)
   yes

   (c) Transform the above subsumption check to a concept satisfiability check for a concept, call it C_0.

$$C_0 = \exists \mathsf{hC}.(\mathsf{B} \sqcap (\geqslant 1\,\mathsf{hF})) \sqcap \exists \mathsf{hC}.(\mathsf{B} \sqcap \forall \mathsf{hF}.\bot) \sqcap (\leqslant 2\mathsf{hC}) \sqcap \neg \forall \mathsf{hC}.\mathsf{B}.$$

   (d) Produce the negation normal form of C_0, call it C_1.

$$C_1 = \exists \mathsf{hC}.(\mathsf{B} \sqcap (\geqslant 1\,\mathsf{hF})) \sqcap \exists \mathsf{hC}.(\mathsf{B} \sqcap \forall \mathsf{hF}.\bot) \sqcap (\leqslant 2\mathsf{hC}) \sqcap \exists \mathsf{hC}.\neg \mathsf{B}.$$

The following tasks are *optional*

4. Regarding task 1: Is it possible to change the meaning of $\mathsf{Optimist}$ in interpretation $\mathcal{I}$ so that (3) does **not** hold? If so, provide such a meaning for $\mathsf{Optimist}$, i.e. the set of optimists in the changed interpretation.

   Yes. If, for example, $\mathsf{Optimist}$ has the following meaning, then axiom (3) does not hold:

$$\mathsf{Optimist}^{I} = \{\mathtt{Steve}\}$$

5. Regarding task 1: Show that the axiom (4) below, is a consequence of $\mathcal{T}$, as defined by (1)–(3).

$$\mathsf{Father} \quad \sqsubseteq \quad \mathsf{Optimist} \sqcup \exists \mathsf{hasChild}.(\mathsf{Optimist} \sqcup \neg \mathsf{Female}). \tag{4}$$

   Hint: show that a counterexample for (4) is a counterexample for (3) as well.

   Consider an arbitrary interpretation $\mathcal{I}'$ in which (4) does not hold. This means that there exists an $a \in \Delta^{\mathcal{I}'}$ such that $a \in \mathsf{Father}^{\mathcal{I}'}$, but $a \notin (\mathsf{Optimist} \sqcup \exists \mathsf{hasChild}.(\mathsf{Optimist} \sqcup \neg \mathsf{Female}))^{\mathcal{I}'}$.
   This means that

$$a \quad \in \quad (\neg \mathsf{Optimist} \sqcap \forall \mathsf{hasChild}.(\neg \mathsf{Optimist} \sqcap \mathsf{Female}))^{\mathcal{I}'} \tag{5}$$

   From (5) it follows that a's all children are female. This, together with a being a father, means $a \in \mathsf{FatherOfGirls}^{\mathcal{I}'}$, i.e. a belongs to the set that represents the left hand side of (3) in $\mathcal{I}'$. At the same time (5) also means that neither a is an optimist nor a has any children who are optimists, which means that a does not belong to the right hand side of (3), thus (3) does not hold in $\mathcal{I}'$.
   Hence in any interpretation in which (3) holds, (4) has to hold as well.
   (Because, if (4) did not hold, then, as we have just shown, (3) would not hold either.)

6. Regarding task 3: Justify your answer to question (b).

   We know x has a child who is blonde and who has at least 1 friend – call her y.
   We also know x has a child who is blonde and who has no friends – call her z.
   y and z cannot be the same person, because the numbers of friends of y and z are different.
   So we know about two (distinct) children of x, both being blonde.
   We also know that x has at most 2 children.
   This means y and z are the only children of x.
   Hence all children of x are blonde.