

This is an open book test, you can use the printed version of the slides and/or the textbook.

1. Consider the interpretation $\mathcal{I} = \langle \Delta, I \rangle$, where

$$\begin{aligned} \Delta &= \{\text{Nick, Mary, Ann, Steve, John, Paul}\} \\ \text{Female}^I &= \{\text{Mary, Ann}\} \\ \text{hasChild}^I &= \{\langle \text{Nick, Ann} \rangle, \langle \text{Mary, Ann} \rangle, \\ &\quad \langle \text{Ann, John} \rangle, \langle \text{Steve, John} \rangle, \langle \text{Ann, Paul} \rangle, \langle \text{Steve, Paul} \rangle\} \\ \text{BlueEyed}^I &= \{\text{Nick, Steve, Ann, Paul}\} \end{aligned}$$

Consider also the TBox \mathcal{T} consisting of the following axioms:

$$\begin{aligned} \text{Grandchild} &\equiv \exists \text{hasChild}^- . \exists \text{hasChild}^- . \top & (1) \\ \text{Grandchild} \sqcap \text{BlueEyed} &\sqsubseteq \forall \text{hasChild}^- . \forall \text{hasChild}^- . \text{BlueEyed} & (2) \end{aligned}$$

- (a) Assuming that axiom (1) holds in \mathcal{I} , which set corresponds to the concept **Grandchild** in \mathcal{I} ? (5 p)
- (b) For both the left and the right hand side of (2) provide an English phrase describing those belonging to the given concept. As an example, such a phrase for the right hand side of (1) could be: “those having a parent who has a parent” or “those having a grandparent”. (10 p)
- (c) Provide the meaning for both the left and the right hand side of (2), i.e. give subsets of Δ that correspond to these two concept expressions in \mathcal{I} . (10 p)
- (d) Decide if (2) holds in \mathcal{I} . (5 p)
- (Σ 30 p)
2. Transform the following sentences into a *SHIQ* TBox using the concepts and roles written in **grotesque** font. Anything in parentheses is explanation only, and should not be formalized.
- (Our universe contains **Humans** and **Robots**). Each human is a **Creature**, and each robot is also a creature (*a-b*). No one can be both a human and a robot (*c*). **Kimaks** are a subgroup of humans (*d*). (Relationships **hasEnemy** and **hasEmployer** are defined. Role **hasEnemy** is not necessarily symmetrical, i.e. from *a* has enemy *b* it does not necessarily follow that *b* has enemy *a*.) A human cannot be an employer of a robot (*e*), and robots have no enemies (*f*). (6*4 p)
- A **Tyrant** is defined as someone who is a kimak, has at least two enemies who are non-kimak creatures, and who has a robot employer (*g*). (8 p)
- Give at least one non-trivial re-formulation (at least two formulations altogether) for statements (*e*) and (*f*). $C \sqsubseteq \perp$ vs. $C \equiv \perp$ and $\top \sqsubseteq C$ vs. $\top \equiv C$ are considered trivial re-formulations. (2*4 p)
- (Σ 40 p)
3. Consider the following subsumption check, with respect to an empty TBox:

$$\exists hC.C \sqcap \exists hC.B \sqsubseteq \exists hC.(C \sqcap B) \sqcup \forall hC.(C \sqcup B) \sqcup (\geq 3hC). \quad (3)$$

- (a) Rewrite the above axiom into a somewhat formalized English sentence (hC stands for “has child”, C stands for “clever” and B stands for “blonde”). Use the following format: “If $x \dots$ then either $x \dots$ ”. (10 p)
- (b) Transform the above subsumption check to a concept satisfiability check for a concept, call it C_0 . (5 p)
- (c) Produce the negation normal form of C_0 , call it C_1 . (10 p)
- (d) Does subsumption (3) hold? (That is, is it true in all interpretations?) A “yes” or “no” answer is enough. (5 p)
- (Σ 30 p)

The following tasks are optional

4. Re task 1: Change the meaning of **BlueEyed** in interpretation \mathcal{I} so that your answer to subquestion (d) changes to its opposite (i.e. if the answer was “yes”, it changes to “no” and if the answer was “no”, it changes to “yes”). (6 p)
5. Re task 1: Show that the axiom (4) below, is a consequence of the TBox \mathcal{T} containing the axioms (1) and (2). (12 p)

$$\text{BlueEyed} \sqsubseteq \forall \text{hasChild}^- . \forall \text{hasChild}^- . \text{BlueEyed} \quad (4)$$

Hint: show that an *arbitrary* counterexample for (4) is a counterexample for (2) as well.

6. Regarding task 3: Justify your answer to subquestion 3 (d). (12 p)
- (Σ 30 p)