Numerical methods of linear algebra
Problems for the exam
2019

~~1.~~ Apply the Gauss–Seidel method to solve the linear system

$$\begin{pmatrix} 7 & 0 & 1 & -1 \\ -1 & 1 & 8 & 0 \\ 0 & 10 & -1 & 1 \\ 10 & 1 & 0 & 30 \end{pmatrix} \cdot x = \begin{pmatrix} 1.2 \\ -8.3 \\ 2.6 \\ 22.1 \end{pmatrix}$$

2. Apply the Gauss–Seidel method to solve the linear system

$$\begin{pmatrix} 8 & 1 & 1 & -1 \\ -2 & 12 & -1 & 0 \\ 2 & 0 & 16 & 2 \\ 0 & 1 & 2 & -20 \end{pmatrix} \cdot x = \begin{pmatrix} 18 \\ -7 \\ 54 \\ -14 \end{pmatrix}$$

~~3.~~ Apply the Gauss–Seidel method to solve the linear system

$$\begin{pmatrix} 6.03 & 3.01 & 1.99 \\ 3.01 & 4.16 & -1.23 \\ 1.99 & -1.23 & 9.34 \end{pmatrix} \cdot x = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

~~4.~~ Use the gradient method to solve the system:

$$\begin{pmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{pmatrix} \cdot x = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 6 \end{pmatrix}$$

~~5.~~ Use the conjugate gradient method to solve the system:

$$\begin{pmatrix} 2 & -1 & 0 & -1 & 0 \\ -1 & 3 & -1 & 0 & -1 \\ 0 & -1 & 2 & -1 & 0 \\ -1 & 0 & -1 & 3 & -1 \\ 0 & -1 & 0 & -1 & 3 \end{pmatrix} \cdot x = \begin{pmatrix} -4 \\ -3 \\ 0 \\ 3 \\ 9 \end{pmatrix}$$

~~6.~~ Compute the solution using the formulas given for the tridiagonal case.

$$\begin{pmatrix} 5 & -4 & 1 & 0 & 0 & 0 \\ -4 & 6 & -4 & 1 & 0 & 0 \\ 1 & -4 & 6 & -4 & 1 & 0 \\ 0 & 1 & -4 & 6 & -4 & 1 \\ 0 & 0 & 1 & -4 & 6 & -4 \\ 0 & 0 & 0 & 1 & -4 & 6 \end{pmatrix} \cdot x = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

~~7.~~ Compute the solution of the following linear system.

$$\begin{pmatrix} 4 & -1 & 0 & -1 & 0 & 0 \\ -1 & 4 & -1 & 0 & -1 & 0 \\ 0 & -1 & 4 & 0 & 0 & -1 \\ -1 & 0 & 0 & 4 & -1 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 \\ 0 & 0 & -1 & 0 & -1 & 4 \end{pmatrix} \cdot x = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

~~8.~~ Use power iteration to approximate the eigenvalues with largest absolute value and compute the corresponding eigenvector.

$$\begin{pmatrix} 1 & 2 & 0 \\ -2 & 1 & 2 \\ 1 & 3 & 1 \end{pmatrix}$$

9. Use power iteration to approximate the eigenvalues with the largest absolute value and compute the corresponding eigenvector.

$$\begin{pmatrix} 7 & -4 & 2 \\ 16 & -9 & 6 \\ 8 & -4 & 5 \end{pmatrix}$$

10. Use power iteration to approximate the eigenvalues with the largest absolute value and compute the corresponding eigenvector.

$$\begin{pmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{pmatrix}$$

11. Compute the eigenvalues with the largest absolute value of the following matrix and find the corresponding eigenvector.

$$\begin{pmatrix} 2 & 0 & 1 & 0 \\ 0 & 0 & 3 & 1 \\ 1 & 3 & 4 & -2 \\ 0 & 1 & -2 & 0 \end{pmatrix}$$

12. Compute the eigenvalue with the largest absolute value for this matrix and find the corresponding eigenvector.

$$\begin{pmatrix} 10 & 1 & 2 & 3 \\ 1 & 9 & -1 & 2 \\ 2 & -1 & 7 & 3 \\ 3 & 2 & 3 & 12 \end{pmatrix}$$

13. Compute the eigenvalue with the largest absolute value for this matrix and find the corresponding eigenvector.

$$\begin{pmatrix} 10 & 1 & 2 & 3 & 4 \\ 1 & 9 & -1 & 2 & -3 \\ 2 & -1 & 7 & 3 & -5 \\ 3 & 2 & 3 & 12 & -1 \\ 4 & -3 & -5 & -1 & 15 \end{pmatrix}$$

14. For this matrix compute its eigenvalue that is in the interval $(15, 17)$.

$$\begin{pmatrix} 5 & 1 & -2 & 0 & -2 & 5 \\ 1 & 6 & -3 & 2 & 0 & 6 \\ -2 & -3 & 8 & -5 & -6 & 0 \\ 0 & 2 & -5 & 5 & 1 & -2 \\ -2 & 0 & -6 & 1 & 6 & -3 \\ 5 & 6 & 0 & -2 & -3 & 8 \end{pmatrix}$$

~~15.~~ Find the eigenvalues and eigenvectors of this matrix.

$$\begin{pmatrix} 3 & -1 & 2 & 7 \\ 1 & 2 & 0 & -1 \\ 4 & 2 & 1 & 1 \\ 2 & -1 & -2 & 2 \end{pmatrix}$$

~~16.~~ Transform A to tridiagonal form and compute its eigenvalues and eigenvectors.

$$A = \begin{pmatrix} 5 & 1 & -2 & 0 & -2 & 5 \\ 1 & 6 & -3 & 2 & 0 & 6 \\ -2 & -3 & 8 & -5 & -6 & 0 \\ 0 & 2 & -5 & 5 & 1 & -2 \\ -2 & 0 & -6 & 1 & 6 & -3 \\ 5 & 6 & 0 & -2 & -3 & 8 \end{pmatrix}$$

17. Transform A to tridiagonal form and compute its eigenvalues and eigenvectors.

$$A = \begin{pmatrix} 120 & 80 & 40 & 16 \\ 80 & 120 & 16 & -40 \\ 40 & 16 & 120 & -80 \\ 16 & -40 & -80 & 120 \end{pmatrix}$$

~~18.~~ Transform A to tridiagonal form and compute its eigenvalues and eigenvectors.

$$A = \begin{pmatrix} 10 & 1 & 2 & 3 & 4 \\ 1 & 9 & -1 & 2 & -3 \\ 2 & -1 & 7 & 3 & -5 \\ 3 & 2 & 3 & 12 & -1 \\ 4 & -3 & -5 & -1 & 15 \end{pmatrix}$$

~~19.~~ Transform A to tridiagonal form and compute its eigenvalues and eigenvectors.

$$A = \begin{pmatrix} 6 & 4 & 1 & 1 \\ 4 & 6 & 1 & 1 \\ 1 & 1 & 5 & 2 \\ 1 & 1 & 2 & 5 \end{pmatrix}$$

~~20.~~ Apply QR transformation to compute the eigenvalues of this matrix.

$$\begin{pmatrix} 2 & 1 & 6 & 3 & 5 \\ 1 & 1 & 3 & 5 & 1 \\ 0 & 3 & 1 & 6 & 2 \\ 0 & 0 & 2 & 3 & 1 \\ 0 & 0 & 0 & 2 & 2 \end{pmatrix}$$

21. Apply QR transformation to compute the eigenvalues of this matrix.

$$\begin{pmatrix} 5 & -2 & -5 & -1 \\ 1 & 0 & -3 & 2 \\ 0 & 2 & 2 & -3 \\ 0 & 0 & 1 & -2 \end{pmatrix}$$

22. Transform the matrix to Hessenberg matrix and apply QR transformation to obtain the eigenvalues.

$$\begin{pmatrix} 7 & 8 & 6 & 6 \\ 1 & 6 & -1 & -2 \\ 1 & -2 & 5 & -2 \\ 3 & 4 & 3 & 4 \end{pmatrix}$$

23. Transform the matrix to Hessenberg matrix and apply QR transformation to obtain the eigenvalues.

$$\begin{pmatrix} 2 & 95 & -38 & 18 & 5 \\ 1 & 47 & -19 & 8 & 1 \\ 2 & 151 & -69 & 28 & 4 \\ -1 & 218 & -88 & 34 & 6 \\ 0 & -208 & 84 & -34 & -5 \end{pmatrix}$$

24. Transform the matrix to Hessenberg matrix and apply QR transformation to obtain the eigenvalues.

$$\begin{pmatrix} 3 & 1 & 2 & 5 \\ 2 & 1 & 3 & 7 \\ 3 & 1 & 2 & 4 \\ 4 & 1 & 3 & 2 \end{pmatrix}$$

25. Transform the matrix to Hessenberg matrix and apply QR transformation to obtain the eigenvalues.

$$\begin{pmatrix} 1 & 2 & 3 & 5 \\ 2 & 4 & 1 & 6 \\ 1 & 2 & -1 & 3 \\ 2 & 0 & 1 & 3 \end{pmatrix}$$

~~26.~~ Apply the method of Lanczos to approximate eigenvalues.

$$A = \begin{pmatrix} 5 & 1 & -2 & 0 & -2 & 5 \\ 1 & 6 & -3 & 2 & 0 & 6 \\ -2 & -3 & 8 & -5 & -6 & 0 \\ 0 & 2 & -5 & 5 & 1 & -2 \\ -2 & 0 & -6 & 1 & 6 & -3 \\ 5 & 6 & 0 & -2 & -3 & 8 \end{pmatrix}$$

~~27.~~ Apply the method of Lanczos to approximate eigenvalues.

$$A = \begin{pmatrix} 10 & 1 & 2 & 3 & 4 \\ 1 & 9 & -1 & 2 & -3 \\ 2 & -1 & 7 & 3 & -5 \\ 3 & 2 & 3 & 12 & -1 \\ 4 & -3 & -5 & -1 & 15 \end{pmatrix}$$

~~28.~~ Solve the least squares problem for

$$\begin{pmatrix} 1 & 1 & 1 & 2 \\ 2 & -2 & 3 & 1 \\ 1 & 3 & 4 & 0 \\ 5 & 1 & 1 & 0 \\ 2 & 4 & 3 & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix} \cdot x = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$