Numerical methods of linear algebra
Exam topics

1. Vector norms: definition, examples, properties.
2. Matrix norms: definition, examples. Matrix norms induced by vector norms. Self-adjoint matrices. Rayleigh quotient.
3. Spectral radius. Gershgorin circle theorem. Norm estimates.
4. Singular value decomposition. Moore-Penrose inverse.
5. System of linear equations. Error of solution and the condition number of the matrix.
6. Solution of a linear system by rank one decomposition. Special case: tridiagonal matrix. Iterative improvement of approximate solutions.
7. Gauss–Seidel algorithm. Method of successive over-relaxation, its convergence. The optimal value of ω (for the special case we covered).
8. Tensor product of matrices. A special differential equation, the Poisson equation. Its discretization, the corresponding (approximate) linear system, and its solution.
9. Gradient method.
10. Conjugate gradient method.
11. Eigenvalue approximation: the power iteration and inverse iteration, theorem of Mises.
12. Transformation of symmetrical matrices to tridiagonal form: Householder transformation. Bidiagonal matrices and singular value computation.
13. Sturm sequence of polynomials, theorem of Sturm. Eigenvalues of symmetrical tridiagonal matrices.
14. Eigenvalue computation for non-symmetrical matrices: QR algorithm. QR algorithm for Hessenberg matrices.
15. Transformation to Hessenberg matrix. The advantage of shifting the matrix before QR transformation.
16. Courant-Fischer theorem and perturbation estimates for eigenvalues: Weyl's theorem, eigenvalues after a rank one change, Interlacing property. Theorem of Wielandt and Hoffman.
17. Approximation of eigenvalues by Lanczos method.
18. The least squares problem, the equivalent linear system and its solution.