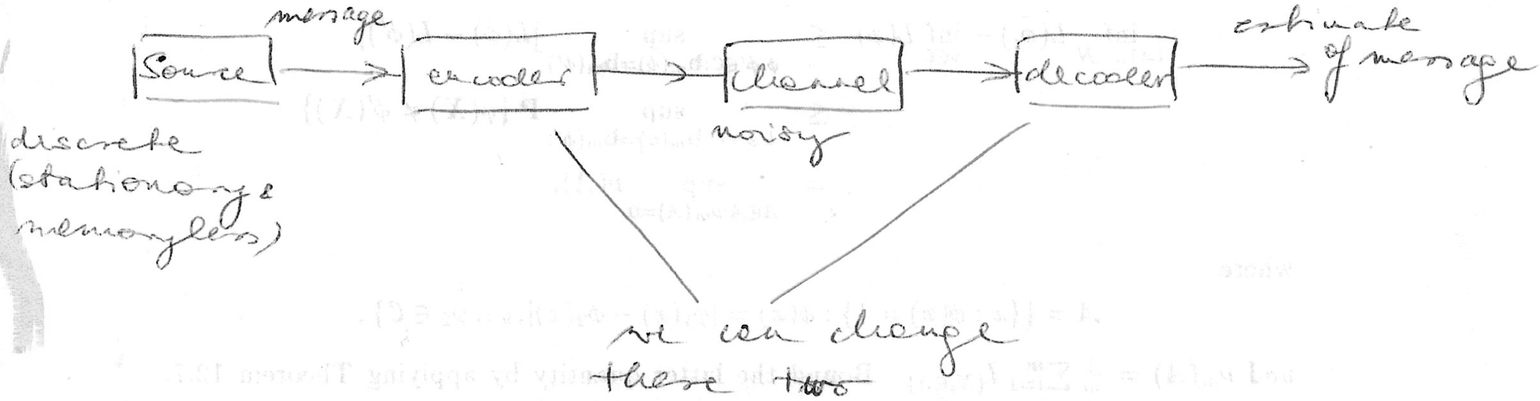
Channel coding

Redundancy → increase the reliability of data transmission

transfer information on a noisy channel



Communication is successful if the receiver and the transmitter agree on what was sent

Source symbols from some finite alphabet are mapped into some sequence of channel symbols, which then produces the output sequence of the channel.

random, but has a distribution that depends on the input seq.

From the output seq, we attempt to recover the transmitted message

Definition

We define a <u>discrete channel</u> to be a system consisting of an input alphabet U, output alphabet V and the transition probabilities $p(\underline{v} \mid \underline{u})$

express the prob. of observing seq. $\underline{v}$ given that seq. $\underline{u}$ was sent

The channel is memoryless

(discrete and memoryless channel DMC)

if for all n and for ~~all~~ any $\underline{u} \in U^n$ and $\underline{v} \in V^n$

$$p(\underline{v} \mid \underline{u}) = \prod_{i=1}^{n} p(v_i \mid u_i)$$

/the prob. distr. of the output depends only on the input at that time and is conditionally independent of previous channel inputs or outputs/

In that case the channel can be described by a channel matrix / prob. transition matrix

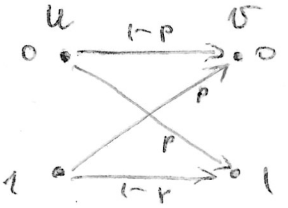
$|U| \times |V|$

$$\begin{array}{c c} & j \\ i & \begin{bmatrix} - & - & p(v_j \mid u_i) \\ & & \\ & & \end{bmatrix} \end{array}$$

Examples

1) binary symmetric channel (BSC)

$U = V = \{0,1\}$



input symbols are complemented with prob p.

simplest model of a channel with errors

in a sense all the received bits are unreliable

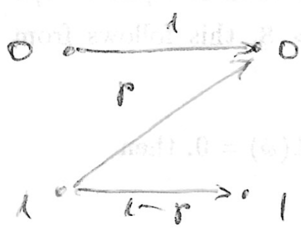
But later we will see that such a communication channel can still be used to send info at a non zero rate with an arbitrarily small prob of error.

matrix

$$\begin{bmatrix} 1-p & p \\ p & 1-p \end{bmatrix}$$

$Pr\{v=0 \mid u=1\} = p$

2) binary Z channel

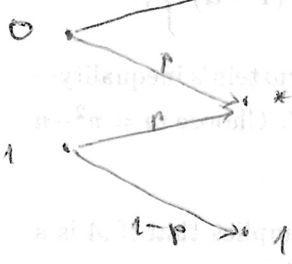


$$\begin{bmatrix} 1 & 0 \\ p & 1-p \end{bmatrix}$$

3) binary erasure channel (BEC)

some bits are lost (rather than corrupted)
the receiver knows which bits have been erased

$u = \{0, 1\}$ $\quad v = \{0, 1, *\}$



$$\begin{array}{c|ccc} & 0 & * & 1 \\ \hline 0 & 1-p & p & 0 \\ 1 & 0 & p & 1-p \end{array}$$