Name: _____

1.	2.	3.	4.	5.	6.	7.	8.	$\sum$
6/	6/	7/	6/	7/	6/	7/	5/	50___

Mathematics II. (BSc)– Exam 3.
Jun 11, 2014.

You need reach at least 20 points to pass.

1. (6 p.) For which values of λ will the following three vectors make a linearly

   independent set?

$$\underline{a}_1 = (\lambda, -1, 1), \quad \underline{a}_2 = (-1, \lambda, 1), \quad \underline{a}_3 = (-1, 1, \lambda)$$

2. (6 p.) Find the values of the parameter a (if possible) such that the following system

   have infinitely many solutions. Solve the system for these a values:

$$\begin{aligned} 2x_1 + \ \ x_2 + ax_3 &= 0 \\ x_1 + ax_2 + 2x_3 &= 0 \\ ax_1 + 2x_2 + \ \ x_3 &= 0 \end{aligned}$$

3. (7 p.) Solve the next differential equation using Laplace transform:

$$y'' + y = \sin 2t, \qquad y(0) = 0, \qquad y'(0) = 1.$$

4. (6 p.) Solve the following differential equation:

$$y' - \frac{4}{x}y = x^4 e^{-2x}, \qquad x \neq 0.$$

5. (7 p.) a.) Find the values of the integral:

$$\iint\limits_{T:\ 1\leq x^2+y^2\leq 9,\ x\leq 0,\ y\geq 0} \sqrt{2x^2+2y^2+5}\,dT.$$

b.) Sketch the region of integration, reverse the order of integration, and evaluate the integral:

$$\int\limits_{x=0}^{2} \int\limits_{y=\frac{x}{2}}^{1} e^{y^2}\,dydx.$$

6. (6 p.) Let the function

$$f(x,y) = e^x(y+1)^2 + (x-1)\cos y$$

is an equation of a surface.

a.) Find the gradient of the function at $P_0\,(0,0)$?

b.) Give the equation of the tangent plane at $P_0\,(0,0)$.

c.) Calculate the directional derivative of $f(x,y)$ at $P_0\,(0,0)$ in the direction

$$\underline{v} = (1,2).$$

7. (7 p.) Given the function $f(x,y) = (2x+y-4)^2 + (x+y)^2$ and a closed triangular

plate bounded by the lines $x = 0, \quad y = 0, \;\; x+y = 1$. Find the absolute

maximum and minimum values of f on the given domain.

8. (5 p.) Find Taylor series at $x_o = -1$ for the function

$$f(x) = \frac{1}{2x+5}$$

and give the radius of the convergence.