Name: _____

Calculus

Lin.Alg.

1.	2.	3.	4.	$\sum$	5.	6.	7.	8.	$\sum$
7/	6/	5/	7/	25___	5/	7/	5/	8/	25___

Mathematics II. (BSc)– 2nd Midterm Test
9th of May, 2013.

1. **Calculus examples**

   (You need reach at least 8 points to pass this part.)

2. (7 p.) Solve the following Cauchy problems:

   a.)

   $$y'' + 4y' + 4y = 0, \quad y(0) = 2, \quad y'(0) = 1;$$

   b.)

   $$xy' + y = \sin x, \quad y(\frac{\pi}{2}) = \frac{1}{\pi}.$$

3. (6 p.) Given the function $f(x, y) = e^{y^2-x-1}(2x + 1)^5$ and a point $P_o(-1, 0)$.

   a.) Find the derivative of f at P_o in the direction of $\underline{v} = -3\underline{i} + 4\underline{j}$.

   b.) Find the direction in which f increases or decreases most rapidly at P_o. Then

   find the derivatives of f in these directions.

   c.) Find an equation for the tangent plane at the point P_o on the given surface.

4. (5 p.) Given the function $f(x, y) = x^3 + y^3 - xy - 4$. Find the maximum and

minimum values of f .

5. (7 p.) Sketch the region of integration, reverse the order of integration, and evaluate

the integral

$$\int_0^{16} \int_{\frac{\sqrt{y}}{2}}^{2} \sqrt[5]{1 + x^3} dy dx.$$

Linear Algebra examples

(You need reach at least 8 points to pass this part.)

6. (5 p.)

$$\underline{\underline{A}} = \begin{pmatrix} 1 & 4 & 3 & 2 \\ -2 & -8 & -6 & -4 \\ 0 & 1 & 7 & 11 \end{pmatrix}, \quad \underline{b} = \begin{pmatrix} 8 \\ -16 \\ 8 \end{pmatrix}$$

a.) Give the rank of $\underline{\underline{A}}$!

b.) Prove that $\underline{A} \cdot \underline{x} = \underline{b}$ solvable!

c.) How many solutions exists?

7. (7 p.)

$$\underline{\underline{A}} = \begin{pmatrix} 3 & 4 \\ 3 & 2 \end{pmatrix}, \quad \underline{b}_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad \underline{b}_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad \underline{b}_3 = \begin{pmatrix} 8 \\ -1 \\ -1 \end{pmatrix}$$

a.) Give the eigenvalues and eigenvectors of $\underline{\underline{A}}^{-1}$ and $\underline{\underline{A}}^2$.
b.) Prove, that $\{\underline{b}_1, \underline{b}_2, \underline{b}_3\}$ form a basis in $\mathbb{R}^3$.
c.) Construate an $\{\underline{c}_1, \underline{c}_2, \underline{c}_3\}$ orthonormal basis, where $\underline{c}_1$ is paralell with $\underline{b}_1$.

8. (5 p.) Which are subspaces of $\mathbb{R}^3$? Please, motivate your answer!

$$A = \left\{ \begin{pmatrix} 0 \\ x \\ y \end{pmatrix}; \quad x, y \in \mathbb{R} \right\}, \qquad B = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix}; \quad 2x + 1 = 3y = 4z + 5 \right\},$$

$$C = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix}; \quad 5x + 4y - z = 0 \right\}, \quad D = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix}; \quad x^2 + y^2 + z^2 = 0 \right\},$$

$$E = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix}; \quad x \geq 0, y, z \in \mathbb{R} \right\}.$$

9. (8 p.)
   a.) Give the $\mathcal{L}\{f(t)\}$ Laplace transformation! $f(t) = t^2 \cos 2t$
   b.) Solve the next differential equation with Laplace transformation:

$$y\prime + 7y = 6, \qquad y(0) = 0.$$