Analysis of matrices

Exam topics, 2018

1. Basic properties, operations of matrices, determinant, rank.
2. Rank one matrices. Minimal rank one decompoition: algorithms, applications (properties of rank).
3. Inverse of a matrix, adjoint matrix, Sherman-Morrison formula.
4. Inverse of special matrices (diagonal, triangular, permutation matrix). Elementary transformations and their matrix forms. Sylvester's law of nullity.
5. Projections, theorem of Egerváry. Biorthogonal system and how to make it a complete system.
6. Types of generalized inverse (generalized, reflexive, Moore-Penrose) and their properties.
7. Special matrices: symmetric, tridiagonal, uniform tridiagonal, Toeplitz, upper Toeplitz, nilpotent. Chebishev polynomials.
8. Determinant, adjoint and inverse of a symmetric uniform tridiagonal matrix.
9. Block matrix (hypermatrix). Symmetric 2 by 2 partition, determinant, inverse expressed by the blocks.
10. Persymmetrical partition and its application to the inverse computation of symmetric tridiagonal and one-pair matrices.
11. Solutions of linear system of equations (homogeneous, inhomogeneous).
12. Linear maps, similar matrices. Eigenvalues, eigenvectors, eigenspaces. Characteristic polynomial. Diagonalizable matrix.
13. Spectral decomposition. Eigenvalues of special matrices (Hermitian, skew Hermitian, unitary, projection).
14. Theorem of Schur. A can be diagonalized by unitary matrix iff A is normal,
15. Singular values, singular value decomposition.
16. Cayley-Hamilton theorem. Minimal polynomial, reduced adjoint.
17. Connection between the minimal polynomial and the property that the matrix can be diagonalized.
18. Functions of matrices. Lagrange polynomials. Properties of matrix $L_j(A)$.
19. Spectral dcomposition of $f(A)$. Special cases: projections and cyclic matrices.
20. Multiple roots of minimal polynomial. Hermite interpolation.
21. Jordan form of matrices.
22. System of linear differential equations (homogeneous, inhomogeneous). Case of constant coefficients.