Sample exercises for the exam of Analysis of Matrices

1. Compute the characteristic polynomial and a minimal rank one decomposition of the matrix

$$A = \begin{bmatrix} 3 & 1 & -3 \\ -7 & -2 & 9 \\ -2 & -1 & 4 \end{bmatrix}$$

2. Create a complete biorthogonal system from these vectors (if it is possibe).

$$v^T = \begin{bmatrix} 2 & 5 & -3 \end{bmatrix}, \qquad u = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}$$

3. Compute the determinant. adjoint, and inverse of the matrix

$$\begin{bmatrix} 3 & -3 & 3 \\ -1 & 5 & 2 \\ -1 & 3 & 0 \end{bmatrix}$$

4. Compute the characteristic polynomial, the minimal polynomial and the eigenvectors of the following matrix:

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & -3 \\ 0 & 1 & 3 \end{bmatrix}$$

5. Compute the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

6. What is the Moore-Penrose pseudo-inverse of this matrix?

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

7. How do the solutions of this linear system depend on parameter λ?

$$\begin{bmatrix} 2 & 5 & 1 & 3 \\ 4 & 6 & 3 & 5 \\ 4 & 14 & 1 & 7 \\ 2 & -3 & 3 & \lambda \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 4 \\ 7 \end{bmatrix}$$

8. Determine the interpolation polynomials for this matrix (Lagrange or Hermite, whichever can be used).

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & -3 \\ 0 & 1 & 3 \end{bmatrix}$$

9. Compute the matrix e^A when

$$A = \begin{bmatrix} 1 & 1 & -1 \\ -3 & -3 & 3 \\ -2 & -2 & 2 \end{bmatrix}$$

10. Determine the solution of the system of differential equations $\dot{x} = Ax$, $x(0) = x_0$ when

$$A = \begin{bmatrix} 1 & 1 & -1 \\ -3 & -3 & 3 \\ -2 & -2 & 2 \end{bmatrix}$$

11. What is the Jordan normal form of the matrix

$$A = \begin{bmatrix} 1 & 1 & -1 \\ -3 & -3 & 3 \\ -2 & -2 & 2 \end{bmatrix}$$

12. For this nilpotent matrix determine its Jordan normal form.

$$A = \begin{bmatrix} 2 & -1 & 1 & -1 \\ -3 & 4 & -5 & 4 \\ 8 & -4 & 4 & -4 \\ 15 & -10 & 11 & -10 \end{bmatrix}$$