

Numerical methods of linear algebra

Exam topics 2017

1. Absolute and relative error in the basic operations. Finite precision, floating point numbers. Vector norms: definition, examples, properties.
2. Matrix norms: definition, examples. Matrix norms induced by vector norms. Self-adjoint matrices. Rayleigh quotient.
3. Spectral radius. Gershgorin circle theorem. Norm estimates.
4. Singular value decomposition. Moore-Penrose inverse.
5. System of linear equations. Error of solution and the condition number of the matrix.
6. Solution of a linear system by rank one decomposition. Special case: tridiagonal matrix. Iterative improvement of approximate solutions.
7. Gauss–Seidel algorithm. Method of successive over-relaxation, its convergence. The optimal value of ω (for the special case we covered).
8. Tensor product of matrices. A special differential equation, the Poisson equation. Its discretization, the corresponding (approximate) linear system, and its solution.
9. Gradient method.
10. Conjugate gradient method.
11. Eigenvalue approximation: the power iteration and inverse iteration, theorem of Mises.
12. Transformation of symmetrical matrices to tridiagonal form: Householder transformation. Bidiagonal matrices and singular value computation.
13. Sturm sequence of polynomials, theorem of Sturm. Eigenvalues of symmetrical tridiagonal matrices.
14. Eigenvalue computation for non-symmetrical matrices: QR algorithm. QR algorithm for Hessenberg matrices.
15. Transformation to Hessenberg matrix. The advantage of shifting the matrix before QR transformation.
16. Approximation of eigenvalues by Lanczos method.
17. Courant-Fischer theorem and perturbation estimates for eigenvalues: Weyl's theorem, eigenvalues after a rank one change, Interlacing property.
18. Theorem of Wielandt and Hoffman, and its comparison to Weyl's theorem.
19. The least squares problem, the equivalent linear system and its solution.
20. Generalized eigenvalue problem.
21. Linear matrix inequalities (application and some basic mathematical ideas).