

1. A tree on 20 vertices contains (exactly) ten vertices of degree 1 and (exactly) one vertex of degree 10. Show that the tree doesn't contain a vertex of degree 3.
2. The vertex set of the simple graph G is $V(G) = \{1, 2, 3, \dots, 100\}$, and two vertices are adjacent in G if and only if the larger of the two numbers is at least twice as large as the smaller one. Does the graph G have a Hamilton cycle?
3. The simple graph G has 20 vertices and 101 edges.
 - a) Is it possible that the chromatic number of G is 2?
 - b) Is it possible that the chromatic number of G is 3?
4. Let the two vertex classes of the bipartite graph $G(A, B; E)$ be $A = \{a_1, a_2, \dots, a_9\}$ and $B = \{b_1, b_2, \dots, b_8\}$. For each $1 \leq i \leq 9$ and $1 \leq j \leq 8$ let a_i and b_j be adjacent if the entry in the i th row and j th column of the matrix on the right is 1. Determine a maximum matching and a minimum vertex cover in G .

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}$$
5. In a simple graph G on 16 vertices the degree of each vertex is 3, and the maximum number of independent edges is 7. Determine the edge-chromatic number of the graph G .
6. * Each component of the simple graph G on 102 vertices has 3 or 4 vertices. We can transform the graph in one step as follows: first we delete 3 arbitrary edges then we add an arbitrary edge (we don't change the vertex set of G). Show that we cannot get a connected graph from G by iterating this step arbitrarily many times.

Please work on stapled sheets only, including drafts, and submit all of them at the end of the midterm.

Write your name on every sheet you work on, and write your Neptun code and the name of your practice instructor (according to neptun) on the first page.

The details of the solutions must be explained; giving the result only is not worth any points. Notes, calculators or any additional devices cannot be used.

You have 90 minutes to work on the problems. Each of them is worth 10 points. The problem marked with an * is supposed to be more difficult. To obtain a signature you have to achieve at least 24 points.