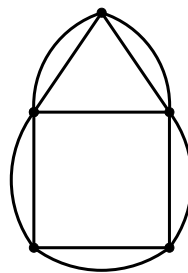


1. A tree on 13 vertices contains two vertices of degree 6. Show that the tree doesn't contain a vertex of degree 3.
2. The vertex set of the simple graph G is $V(G) = \{1, 2, 3, \dots, 8\}$. In G , 1 is adjacent to 8, and beside this the vertices $x, y \in V(G)$, $x \neq y$ are adjacent in G if and only if $|x - y| \leq 2$. At least how many vertices must be deleted from G (together with the edges incident to them) so that the graph obtained contains an Euler trail?
3. All we know about the graph G is that it contains exactly two odd cycles and both of those contain the vertex v .
 - a) Is it possible that the chromatic number of G is 2?
 - a) Is it possible that the chromatic number of G is 3?
 - a) Is it possible that the chromatic number of G is 4?

4. Let the two vertex classes of the bipartite graph $G(A, B; E)$ be $A = \{a_1, a_2, \dots, a_8\}$ and $B = \{b_1, b_2, \dots, b_8\}$. For each $1 \leq i \leq 8$ and $1 \leq j \leq 8$ let a_i and b_j be adjacent if the entry in the i th row and j th column of the matrix on the right is 1. Determine a maximum matching and a maximum independent set of vertices in G .

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

5. Determine the edge-chromatic number of the graph G below.



6. * In a simple, connected graph G on 30 vertices the degree of each vertex is at least 10. We know that G has an edge such that after deleting it we get a non-connected graph. Show that G has a Hamilton path.

Please work on stapled sheets only, including drafts, and submit all of them at the end of the midterm.

Write your name on every sheet you work on, and write your Neptun code and the name of your practice instructor (according to neptun) on the first page.

You have 90 minutes to work on the problems. Each of them is worth 10 points. The problem marked with an * is supposed to be more difficult. To obtain a signature you have to achieve at least 24 points.

The details of the solutions must be explained; giving the result only is not worth any points. Notes, calculators or any additional tools cannot be used.