ITC2+ CGT1 **Midterm Testgrading guide**

General Rules. Disclaimer: Google translate has been used for this section. The purpose of the scoring guide is to ensure that the dissertations are evaluated uni- formly by the correctors. Therefore, the guide the main ideas for solving each task (at least one possible) and the marks assigned to them communicates sub-scores. The guide is not intended to detail the complete solution of the tasks description; the steps described can be considered as a sketch of a solution with a maximum score. The sub-scores indicated in the guide only accrue to the solver if the related idea is included in the dissertation as a step towards a clear, clearly described and justified solution. Thus, for example, stating the definitions and items in the material without knowing how to apply them does not deserve any points (even if any of the facts described are indeed used in the solution). Deciding the score based on the points indicated in the guide in light of the above is under the grader's full remedial authority. A partial score is awarded for each idea or partial solution from which, with a suitable addition, a flawless solution to the problem would have been obtained. If a solver starts several several substantially different solutions for a task, he can be assigned to at most one score. If all the solutions or parts of solutions described are correct or correct, then the solution initiative worth the most subpoints is evaluated. However, if amongst several solution attempts there is a correct solution but also an incorrect one (with a substantial error), and it is not clear from the dissertation which the solver considered as correct, then the solution with fewer points is evaluated (even if this score is 0). The sub-scores in the guide can be further divided if necessary. A good solution other than that described in the guide is, of course, worth a maximum point. Theorems can be stated without proof, but only those discussed in class.

1. We want to make a new password for ourselves for safety reasons. We want the following conditions to be satisfied:
   a) it should consist of letters only (from the 26 letters of the English alphabet),
   b) no letter should appear more than once,
   c) exactly 14 letters should appear, moreover 6 lowercase and 8 uppercase letters, but one letter in only one form.
   How many passwords can we choose from with these conditions?

   Solution:
   If lower- and uppercase doesn't matter yet: (1 point)
   $26 \cdot 25 \cdot \dots \cdot 13$, variations without repetitions (3 points)
   places of uppercase letters: $\binom{14}{8}$ (4 points)
   final answer: $26 \cdot 25 \cdot \dots \cdot 13 \cdot \binom{14}{8}$. (2 points)

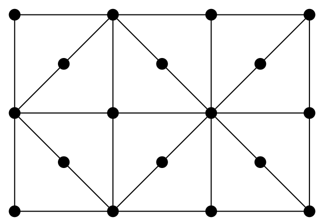
2. We add all the shortest diagonals to a regular 14-sided polygon. Determine the chromatic number of the graph (with 14 vertices and 28 edges) obtained.

   Solution:
   There is a good coloring with 4 colors (3 points)
   3 colors are not enough: by contradiction, (1 point)
   if the vertices are $1, 2, \dots, 14$ (clockwise), then 1, 2 and 3 get different colors (1 point)
   then the color of 4 must be the same as the color of 1, (1 point)
   because... (1 point)
   the color of 5 must be the same as the color of 2,..., the color of 13 must be the same as the color of 1 (1 point)
   a contradiction, because they are adjacent (1 point)

So the chromatic number is at least 4, and then it must be 4. (1 point)

If the assumption by contradiction is missing, then the 7 points for it cannot be given. The greedy coloring in itself is not enough.

3. Determine a maximum matching and a minimum covering set of vertices in the graph below.

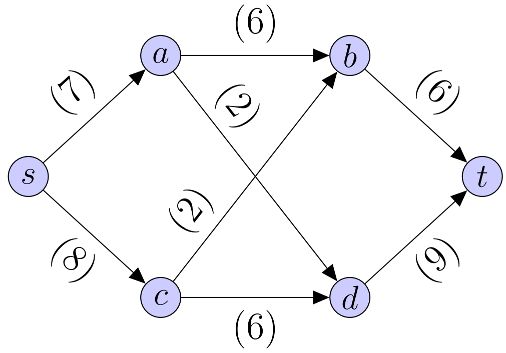


Solution:
There is a matching of size 6. (2 points)
There are 6 covering vertices (every other vertex from the grid). (3 points)
These are maximum and minimum:
We know that $\nu(G) \leq \tau(G)$ holds for each graph. (1 point)
From the matching we have $\nu(G) \geq 6$, (1 point)
and from the covering set of vertices $\tau \leq 6$. (1 point)
So $6 \leq \nu(G) \leq \tau(G) \leq 6$, (1 point)
and $\nu(G) = 6$ and our matching is maximum, and $\tau = 6$ and our covering set of vertices is minimum. (1 point)

4. Is there a complete bipartite graph on 101 vertices which contains an Euler trail? (In a complete bipartite graph all the vertices in one class are connected to all the vertices in the other class.)

Solution:
The complete bipartite graph is $K_{a,b}$ with $a \leq b$. (0 points)
($K_{0,101}$ contains an Euler circuit.) (0 points)
If $a \geq 1$, then $K_{a,b}$ is connected, because there is a path between every pair of vertices: . (1 point)
check 2 cases:... (2 points)
The degrees: there are a vertices of degree b, and b vertices of degree a. (1 point)
Exactly one of a and b odd, since their sum is odd. (2 points)
Euler's theorem: the number of odd-degree vertices must be 0 or 2, and the graph must be connected. (2 points)
If $a = 2$, then $K_{2,99}$ contains an Euler circuit. (2 points)

5. Determine a maximum flow from s to t and a minimum s,t-cut in the network below.



Solution:
There is a flow of value 14. (4 points)
The capacity of the s, t-cut with $X = \{s, a, b, c\}$ is 14, (3 points)
because... (1 point)
We know that $14 \leq \max m(f) = \min c(C) \leq 14$, (1 point)
therefore the flow is maximum and the cut is minimum. (1 point)

6. * We select 30 squares on a 100×100 chessboard in such a way that they form a connected area (i.e. we can get from any of the selected squares to any other one by moving through selected squares with a common side only). Show that we can place eight 1×2 domino pieces without overlap on the selected squares in such a way that each domino covers exactly 2 (neighboring) squares.

Solution:
Construct a graph G, vertices: squares, edges: neighbors. (1 point)
Then G is connected, (1 point)
so it has at least 29 edges. (1 point)
G is bipartite, because... (1 point)
The dominoes correspond to edges of G, (1 point)
and their placement to a matching in G. (1 point)
So we have to prove that $\nu(G) = \tau(G)$ (in bipartite graphs) is ≥ 8. (1 point)
In G, $\Delta(G) \leq 4$, (1 point)
so 7 vertices can cover at most $7 \cdot 4 = 28$ edges, (1 point)
so a set of covering vertices has to contain at least 8 vertices. (1 point)