Abstract

Let $\mathcal{H}$ be a k-uniform hypergraph. A chain in $\mathcal{H}$ is a sequence of its vertices such that every k consecutive vertices form an edge. In 1999 Gyula Y. Katona and Hal Kierstead suggested to use chains in hypergraphs as the generalisation of paths. Although a number of results have been published on Hamilton-chains in recent years, the generalisation of trees with chains has still remained an open area.

We generalise the concept of trees for uniform hypergraphs. We say that a k-uniform hypergraph $\mathcal{F}$ is a hypertree if every two vertices of $\mathcal{F}$ are connected with a chain, and an appropriate kind of cycle-free property holds. An edge-minimal hypertree is a hypertree whose edge set is minimal with respect to inclusion.

After considering these definitions, we show that a k-uniform hypertree on n vertices has at least $n-(k-1)$ edges up to a finite number of exceptions, and it has at most $\binom{n}{k-1}$ edges. The latter bound is asymptotically sharp in the case of $k=3$. We give an upper bound on the edge-number of an edge-minimal hypertree and conjecture that $\frac{1}{k-1}\binom{n}{2}$ is an upper bound. We give a construction to show that if the conjecture is true then the inequality is sharp in asymptotic sense.