Numerical methods of linear algebra
Exam topics 2023

1. Absolute and relative error in the basic operations. Finite precision, floating point numbers. Vector norms: definition, examples, properties.
2. Matrix norms: definition, examples. Matrix norms induced by vector norms.
3. Self-adjoint matrices. Quadratic forms. Rayleigh quotient.
4. Spectral radius. Gershgorin circle theorem. Norm estimates.
5. Singular value decomposition. Moore-Penrose inverse.
6. System of linear equations. Error of solution and the condition number of the matrix.
7. Solution of a linear system by rank one decomposition. Special case: tridiagonal matrix. Iterative improvement of approximate solutions.
8. Gauss–Seidel algorithm. Method of successive over-relaxation, its convergence. The optimal value of ω (for the special case we covered).
9. A special differential equation, the Poisson equation. Its discretization, the corresponding (approximate) linear system, and its solution.
10. Gradient method.
11. Conjugate gradient method.
12. Eigenvalue approximation: the power iteration and inverse iteration, theorem of von Mises.
13. Transformation of symmetrical matrices to tridiagonal form: Householder transformation. Bidiagonal matrices and singular value computation.
14. Sturm sequence of polynomials, theorem of Sturm. Application to the eigenvalues of symmetrical tridiagonal matrices.
15. Eigenvalue computation for non-symmetrical matrices: QR algorithm. QR algorithm for Hessenberg matrices.
16. Transformation to Hessenberg matrix. The advantage of shifting the matrix before QR transformation.
17. Approximation of eigenvalues by Lanczos method.
18. Courent-Fischer theorem and perturbation estimates for eigenvalues: Weyl's theorem, eigenvalues after a rank one change, interlacing property.
19. Theorems of Wielandt-Hoffman, and Weyl.
20. The least squares problem, the equivalent linear system and its solution.
21. Non-negative matrices. Primitive and irreducible matrices. Perron-Frobenius theorem
22. Applications, stochastic matrices.