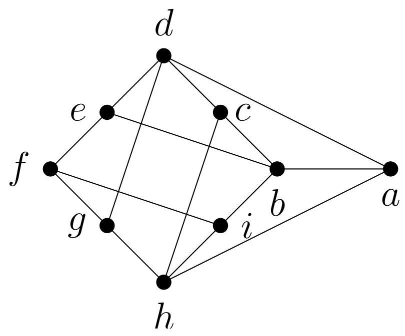
1. Show that there is no such tree on $n \geq 5$ vertices which has only two kinds of degrees, each of them occurring exactly $\frac{n}{2}$ times (n is even).

2. Let the vertices of the graph G be all the 3-element subsets of the 6-element set $S = \{a, b, c, d, e, f\}$, and two vertices be adjacent if and only if the corresponding subsets have at most one element in common. Does this graph G contain an Euler circuit?

3. ($*$) Can we visit each square of a 3×5 chessboard exactly once with a horse? (A horse in one move moves two squares vertically or horizontally and then one more square perpendicularly to the previous direction.)

4. We add two edges to a bipartite graph on 10 vertices. Is it possible (with an appropriate choice of the graph and the added edges) that the chromatic number of the graph obtained is 4?

5. Determine a minimum set of covering vertices in the graph below.



6. We only know of the graph G that it was obtained from $K_{9,9}$ by deleting 8 edges from it. Determine $\chi_e(G)$, the edge-chromatic number of G.

Total work time: 90 min.
The full solution of each problem (including explanations) is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit.
Notes and calculators (and similar devices) cannot be used.