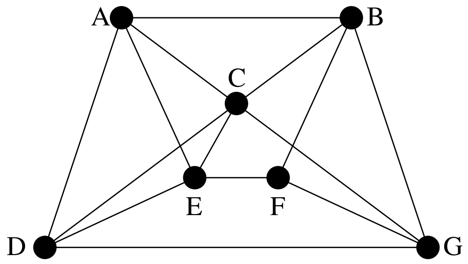
1. Let the vertex set of the simple graph be $V(G) = \{1, 2, \ldots, 10\}$. Let the vertices $x, y \in V(G)$, $x \neq y$ be adjacent if and only if $|x - y| \leq 2$. Does G have a spanning tree, which
a) contains all the edges $\{x, y\}$ of G for which $x, y \leq 3$ holds;
b) contains all the edges $\{x, y\}$ of G for which $|x - y| = 2$ holds?

2. Decide whether the following graph is planar or not. If yes, then draw it with straight edges without crossing; if not, then prove it.

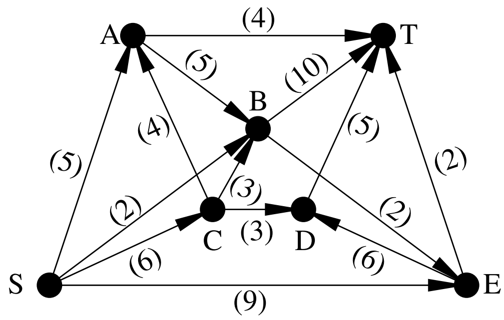


3. ($*$) Can we list all the four-element subsets of the set $\{1, 2, \ldots, 10\}$ in one sequence in such a way that the subsets which are next to each other in the sequence have at least two elements in common (and each subset appears exactly once in the sequence)?

4. Let the vertex set of the simple graph be $V(G) = \{1, 2, \ldots, 10\}$. Let the vertices $x, y \in V(G)$ be adjacent if and only if $|x - y| = 3$ or $|x - y| = 5$. Determine
a) $\chi(G)$, the chromatic number of G;
b) $\chi_e(G)$, the edge-chromatic number of G.

5. Delete the edge $\{3, 8\}$ from the graph defined in Exercise 4, and denote the graph obtained by H. (So the vertex set of H is the same as that of G in Exercise 4, but it has one less edges.)
a) Determine $\nu(H)$, the maximum number of independent edges in H and determine a maximum matching in H.
b) Determine $\alpha(H)$, the maximum number of independent vertices in H and determine a maximum independent set of vertices in H.

6. Determine a maximum flow in the network below (from S to T).



Total work time: 90 min.
The full solution of each problem (including explanations) is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit.
Notes and calculators (and similar devices) cannot be used.