Introduction to Computer Science 5/25/2016

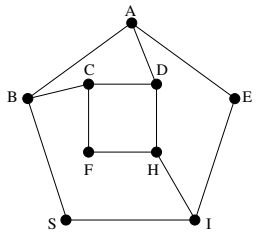
Second Repeat of the First Midterm Test

1. How many sequences of length 4 are there which contain 4 different numbers from $1, 2, \ldots, 100$ such that the largest number is the first one and the others are in increasing order? (E.g. 25,5,10,16 is such a sequence.)

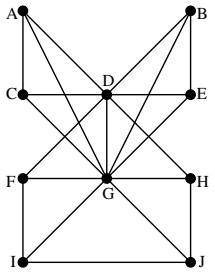
2. Let G be a simple graph and $v \in V(G)$ be a vertex of odd degree. Show that there is a path in G which starts at v and ends in a vertex of odd degree different from v.

3. The graph G doesn't contain a subgraph homeomorphic to $K_{2,3}$ (the complete bipartite graph on 2+3 vertices). Does it follow that G is planar?

4. From the graph below one edge is missing (but all the vertices are there). Can we determine for sure which two vertices were connected by the missing edge if we know that the BFS algorithm started from the vertex S visited the vertices of the graph in the following order:
   a) $S, B, I, C, A, F, H, E, D$;
   b) $S, I, B, E, F, H, C, A, D$?
   In the case where the missing edge can be determined unambiguously, determine the corresponding BFS-tree as well.



5. Does the graph below contain a
   a) Hamilton path,
   b) Hamilton cycle?



6. 11 children play a game. They stand in a circle, and one of them starts passing a ball to somebody else, who in turn passes it on to a third child, etc. The rules are the following: nobody can throw the ball to somebody he/she has thrown it before, also nobody can throw the ball to somebody who has thrown it to him/her before, and nobody can throw the ball to either of the two children standing next to him/her in the circle. At most how many passes are possible in the game under these rules?

Total work time: 90 min.
The full solution of each problem (including explanations) is worth 10 points.
Grading: 0-23 points: 1, 24-32 points: 2, 33-41 points: 3, 42-50 points: 4, 51-60 points: 5.