Introduction to the Theory of Computing 2.

Exercise-set 8.
Solutions

1. When k committees have at least k members together, for $k = 1, 2, \ldots$ (Hall's condition).
2. a) Yes.
   b) No (H, J, L, M like only B, E, F).
3. (a) Count the number of edges between A and B in two ways.
   (b) Count the number of edges between X and $N(X)$ in two ways.
   (c) Frobenius' theorem.
4. Use Hall's condition for (for the people-chocolates bipartite graph) for $|X| \leq n$ and $|X| \geq n+1$, resp.
5. Use Hall's condition for $|X| \leq \frac{n}{2}$ and $|X| \geq \frac{n}{2}$, resp.
6. There is a non-connected counterexample.
7. Can select the edges greedily or use Hall's condition.
8. a) Use Frobenius' theorem.
   b) Use Hall's theorem or unite the vertices of degree 3 and use exercise 3.
9. No perfect matching: $N(\{a_1, a_2, a_4, a_6, a_8\}) = \{b_2, b_3, b_6, b_8\}$.
10. The (rows, colums; coins) bipartite graph is 4-regular.
11. Hall's condition holds for the (figures, sets; containment) bipartite 4-regular (multi)graph.
12. First: $|A| = |B| = 10$, then check Hall's condition.
13. a) $\nu(G) = \tau(G) = 8$, a minimum covering set is $\{a_2, a_3, a_6, a_8, b_1, b_4, b_7, b_9\}$.
    b) $\nu(G) = \tau(G) = 8$, a minimum covering set is $\{a_1, a_3, a_6, a_9, b_1, b_3, b_6, b_8\}$.
14. $\nu(G) = \tau(G) = 100$, $\rho(G) = 102$, a maximum matching e.g. is $\{\{a_i, b_{i+1}\},\ i = 1, 2, \ldots, 100\}$.
15. a) $\nu(G) = \tau(G) = 6$,
    b) $\nu(G) = \tau(G) = 6$,
    c) $\nu(G) = \tau(G) = 9$,
    d) $\nu(G) = \tau(G) = 6$.
16. a) $\nu(G) = \tau(G) = 4$.
    b) $\alpha(G) = 6$.
17. $\nu(G) = \tau(G) = 4,\ \rho(G) = 10$.
18. a) The two endpoints of the edges in the matching can get the same color.
    b) $\overline{G}$ is a regular bipartite graph $\Longrightarrow$ $\omega(G) = 50$ and also $\overline{G}$ contains a perfect matching $\Longrightarrow$ $\chi(G) = 50$.