Introduction to the Theory of Computing 2.

Exercise-set 4.
Solutions

1. a) Yes (2 triangles),
   b) No ($n - e + r = 2$).

2. No ($n - e + 2n = 2 \implies e = 3n - 2$, contradiction).

3. $n = 8,\ r = 10$.

4. $n = 20,\ r = 12,\ k \cdot r = 2e,\ n - e + r = 2 \implies k = 5$ (dodecahedron).

5. No, then $3n \leq e \leq 3n - 6$, contradiction.

6. a) If k vertices have degree 5 and $n - k$ more than 5, then $5k + 6(n - k) \leq 6n - 12 \implies k \geq 12$.
   b) No, e.g: icosahedron.

7. By contradiction, if both G and $\overline{G}$ are planar $\Longrightarrow$ $|E(G)|, |E(\overline{G})| \leq 13 \cdot 3 - 6$, but $|E(G)| + |E(\overline{G}|) = \binom{13}{2}$, contradiction.

8. At most 2: $e \leq 3n - 6$. Adding 2 edges is possible.

9. a) Then $|E| = 3(n-1) > 3n - 6 \Longrightarrow G$ cannot be planar.
   b) Otherwise it contained a spanning tree + a).

10. a) $|E(K_8) = 28 = (3 \cdot 8 - 6) + 10$, and each „additional" edge creates a new crossing with the „planar" ones.
    b) $|E(K_{4,4}) = 16 = (2 \cdot 8 - 4) + 4 \Longrightarrow \exists \geq 4$ edge-crossings.

11. b), f) and k) are planar, the rest are nonplanar.

12. G cannot contain a subgraph homomorphic to K_5 or $K_{3,3}$.

13. Yes, G cannot contain a subgraph homomorphic to K_5 or $K_{3,3}$.

14. a) A nonplanar graph has at least 9 edges.
    b) The complement of a K_5 subgraph contains $K_{3,3}$.

15. The graph is a forest.