Exercise-set 3.
Solutions

1. There are 1, 2 3 an 6 of them, respectively.
2. a) $n_1 \cdot 1 + n_2 \cdot 2 + 5 \cdot 3 = 2(n_1 + n_2 + 5 - 1) \implies n_1 = 7$.
3. One of the degrees is 1. $d \cdot 9 + 92 \cdot 1 = 200 \implies d = 12$.
4. The tree has an even number of vertices.
5. $10(n-1) = \binom{n}{2} - (n-1) \implies n = 1$ or $n = 22$.
6. Necessary: $n - 1 = \binom{n}{2} - (n-1) \implies n = 1$ or $n = 4$. Both are possible.
7. $k + (25 - k) \equiv 2 \cdot 24 \pmod{m} \implies m = 23$
8. a) no;
   b) yes.
9. A graph is a spanning tree and 3 more edges, each of which forms a cycle with the tree.
10. There is a cycle, of lenght at least 3.
11. The number of edges in a spanning forest is 17.
12. A degree one vertex in a spanning tree is like that.
13. a) yes,
    b) no,
    c) no,
    d) yes.
14. a) $S, G, E, A, H, B, F, C, D$.
    b) No.
15. The edge not in the BFS spanning tree started from s whose endpoints are closest to s determines such a cycle.
16. a) no,
    b) yes,
    c) yes.
17. 99 (must be a tree).
18. There are 36 minimum weight spanning trees of weight 19.
19. There are 99! minimum weight spanning trees of weight $2 + 3 + \cdots + 100 = 5049$.
20. The weight of a minimum weight spanning tree is 150.
21. By Kruskal's algorithm: when we get to e, we cannot create a cycle.
22. By Kruskal's algorithm: the other edges of C can be selected before e.
23. The edges of this spanning tree together with another minimum weight spanning tree cannot contain a cycle which contains the edge of weight 100.
24. Order the edges of G such that the edges of the given spanning tree come first among the edges of the same weight.