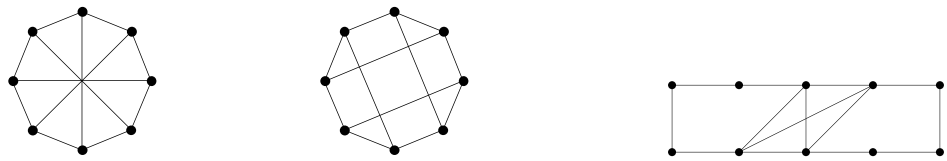
Introduction to the Theory of Computing 2.

Exercise-set 6.

1. Determine whether the first two graphs below are bipartite or not:



2. (MT'09) At least how many edges must be deleted from the third graph above to get a bipartite graph?

3. (MT'10) 7 knights are put on a chessboard in such a way that each of them attacks at least two others. Show that there is such a knight among them which attacks three others.

4. (MT+'16) Let the vertices of the graph G be the all the 0-1 sequences of length 5, and two sequences be adjacent if they differ in eactly one position. Is G a bipartite graph?

5. (MT++'16) Is there a simple bipartite graph on at least 5 vertices whose complement is also a bipartite graph?

6. (MT+'17) In a graph on 99 vertices two vertices have degree 3, and the degree of the other vertices is 4. Show that the graph contains an odd cycle.

7. Determine all the nonisomorphic simple graphs G on 8 vertices for which $\chi(G) = 2$ but if we add any edge to G (between two nonadjacent vertices) then for the graph G' obtained this way $\chi(G') = 3$ holds.

8. (MT+'03) Determine all the nonisomorphic simple graphs G on n vertices for which $\chi(G) = 3$ but if we delete any vertex from G (together with the edges adjacent to it) then for the graph G' obtained $\chi(G') = 2$ holds.

---

9. Determine the chromatic number of the graph of the regular octahedron. (The octahedron has 6 vertices and 8 triangular faces.)

10. Let the vertices of the graph G be the squares of the chessboard, and two vertices be adjacent if and only if the corresponding squares can be reached from each other by one move of a rook. Determine $\chi(G)$, the chromatic number of G. (A rook in chess can move either horizontally or vertically, and in one move it can go to any square along the selected line.)

11. Let the vertices of the graph G be the integers 1,2,...,100, and two vertices, m and n be adjacent if and only if $m + n$ is odd. Determine $\chi(G)$, the chromatic number of G.

12. (MT+'18) We add two non-adjancent edges to the complete bipartite graph $K_{3,3}$ in such a way that the resulting graph G is simple. Determine $\chi(G)$, the chromatic number of G.

13. (MT'05, MT+'15) Determine the chromatic number of the graphs below:

14. (MT'14) Let G be the graph obtained from a regular 11-sided polygon by adding all the shortest diagonals to it (i.e. G has 11 vertices and 22 edges). Determine $\chi(G)$ and $\omega(G)$.

15. Determine the chromatic number of the complement of the cycle on n vertices.

16. Let the vertices of the graph G be the numbers 1,2,...,2015, and two vertices be adjacent if and only if the difference of the corresponding numbers is at most 9. Determine $\chi(G)$, the chromatic number of G.

17. (MT+'16) Let the vertex set of the graph G be $V(G) = \{1, 2, \ldots, 30\}$. Let the vertices $x, y \in V(G)$ be adjacent in G if the difference of the numbers x and y is at least 7. Determine $\chi(G)$, the chromatic number of G.

18. (MT++'03) Let the vertex set of the graph G be $V(G) = \{1, 2, \ldots, 100\}$. Let the vertices $x, y \in V(G)$ be adjacent in G if $x \neq y$ and $100 \leq x \cdot y \leq 400$. Determine the value of $\chi(G)$.

19. (MT++'09) Let the vertices of the graph G be the numbers 1,2,...,15, and two vertices be adjacent if and only if one of the corresponding numbers divides the other. Determine $\chi(G)$, the chromatic number of G.

20. Let the vertices of the graph G be the numbers 1,2,...,30, and two vertices be adjacent if and only if the corresponding numbers are relatively prime. Determine $\chi(G)$, the chromatic number of G.

21. (MT'16) Let the vertex set of the graph G on 9 vertices be the vertices of the unit cube together with the center of it, i.e. $V(G) = \{(x, y, z) : x, y, z \text{ are 0 or 1}\} \cup \{(1/2, 1/2, 1/2)\}$. Let two vertices of G be adjacent if they differ either in the first or the second coordinate, or both. (E.g. $(0, 0, 1)$ is adjacent to $(0, 1, 1)$ and $(1, 1, 0)$ but not to $(0, 0, 0)$.) Determine $\chi(G)$, the chromatic number of G.

22. (MT++'15) In a simple graph G on 10 vertices the degree of each vertex is 8. Determine the chromatic number of G.

23. (MT'19) Let the vertex set of the complete graph K_{10} be $V(K_{10}) = \{1, 2, ..., 10\}$. We obtain the graph G by deleting the edges $\{1, 2\}, \{1, 3\}, \{2, 3\}$ and $\{4, 5\}, \{4, 6\}, \{5, 6\}$ from K_{10}. Determine $\chi(G)$, the chromatic number of G.

24. (MT+'19) We add two edges to a bipartite graph on 10 vertices. Is it possible (with an appropriate choice of the graph and the added edges) that the chromatic number of the graph obtained is 4?

25. (MT++'19) Let the vertices of the graph G be the numbers $1, 2, \ldots, 100$, and two (different) vertices be adjacent if and only if at least one of 2, 3 or 5 is a common divisor of the respective numbers. Determine $\chi(G)$, the chromatic number of the graph G.

---

26. Let the vertex set of the graph G be $V(G) = \{1, 2, \ldots, 2015\}$. Suppose that every vertex of G is adjacent to at most 10 smaller numbers. Prove that $\chi(G) \leq 11$.

27. In the simple graph G apart from 100 exceptional vertices the degree of each vertex is at most 99. Prove that $\chi(G) \leq 100$.

28. (MT++'15) A simple graph G on 10 vertices contains one vertex of degree 5, one of degree 4, one of degree 3, and the rest of the vertices have degree 2. Show that G can be colored with 3 colors.