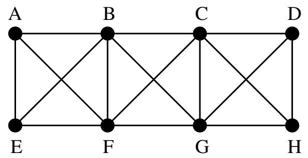
Introduction to the Theory of Computing 2.

# Exercise-set 4.

1. a) Is there a plane graph in which the number of vertices, edges and regions are all divisible by 3?
   b) Is there a connected graph with the above properties?
2. Is there a simple connected plane graph which has half as many vertices as regions?
3. In a simple, connected plane graph the degree of each vertex is 4, and the number of the edges is 16. Determine the number of regions of the drawing.
4. A convex polyhedron has 20 vertices and 12 faces. Each face of the polyhedron is bounded by the same number of edges. What is this common number?
5. Is there a simple planar graph in which the degree of each vertex is at least 6?
6. a) In the simple planar graph $G$ the degree of each vertex is at least 5. Show that in this case $G$ contains at least 12 vertices of degree 5.
   b) Is the statement true with 13 instead of 12?
7. (MT++'05) Let $G$ be a simple planar graph on 13 vertices. Prove that at least one of $G$ and its complement $\overline{G}$ is not planar.
8. (MT'15) At most how many edges can be added to the graph below in such a way that we get a simple planar graph? (We add edges only between already existing vertices.)




9. a) (MT+'05) Suppose that $G = (V, E)$ is a simple graph whose set of edges $E$ is the union of the disjoint sets of edges $E_1$, $E_2$ and $E_3$, where all the three subgraphs $(V, E_1)$, $(V, E_2)$ and $(V, E_3)$ are spanning trees of $G$. Show that in this case $G$ is not planar.
   b) (MT+'18) Let $G$ be a simple, connected planar graph. Show that if we delete the edges of two edge-disjoint spanning trees from $G$ then the remaining graph cannot be connected.
10. a) Show that in a drawing of $K_8$ in which three edges cannot go through a common point there are at least 10 edge-crossings.
    b) At least how many edge-crossings are there in a drawing of $K_{4,4}$ if three edges cannot go through a common point?

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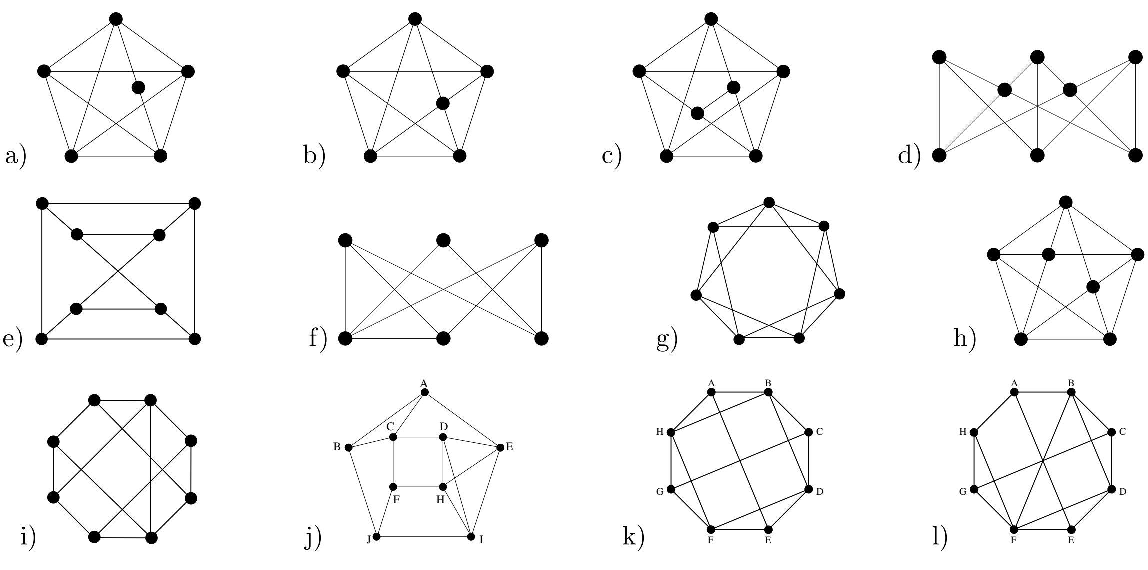
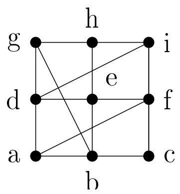
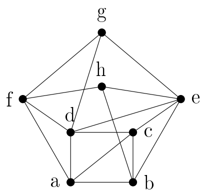
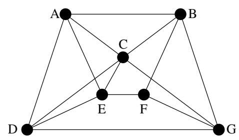
11. Decide whether the following graphs are planar or not. If yes, then draw them with straight edges without crossing; if not, then prove it.



m) MT+'17



n) MT++'17



o) MT'18



12. (MT+'06) In the graph $G$ the degree of each vertex is at most 3. Furthermore we know that each cycle of $G$ contains at most 5 edges. Show that $G$ is a planar graph.

13. (MT++'16) The graph $G$ doesn't contain a subgraph homeomorphic to $K_{2,3}$ (the complete bipartite graph on 2+3 vertices). Does it follow that $G$ is planar?

14. a) Let $G$ be a simple graph on at most 6 vertices. Pove that at least one of $G$ and its complement $\overline{G}$ is planar.
    b) Give a simple graph $G$ on 8 vertices such that neither $G$ nor $\overline{G}$ is planar.

15. (MT'19) A graph on 20 vertices has 18 edges and two components. Show that the graph is planar.