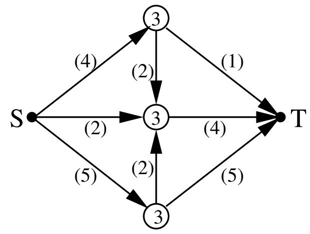
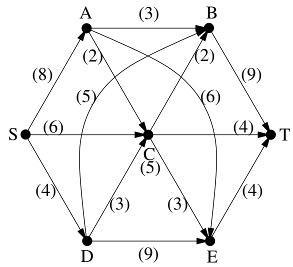
Introduction to the Theory of Computing 2.

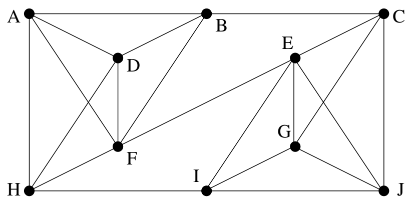
Exercise-set 11.

1. Determine the value of a maximum flow in the networks with edge- and vertex capacities below, and prove that they are maximal.





2. At most how many pairwise edge-disjoint and vertex-disjoint paths are there between the following points in the graph below:
a) B and I, b) A and J, c) B and H.



3. (MT'12) The graph G on 15 vertices is constructed from three cycles, on 4, 5 and 6 vertices each, in such a way that each vertex of the 5-vertex cycle was connected (with one edge) to all the other vertices of the other two cycles. Let s be a vertex of the 4-vertex cycle, and t be a vertex of the 6-vertex cycle.
a) At most how many pairwise vertex-disjoint paths are there in G between s and t?
b) At most how many pairwise edge-disjoint paths are there in G between s and t?

4. Determine the vertex- and edge connectivity numbers ($\kappa(G)$ and $\lambda(G)$) of the following graphs:
a) the graph consisting of the vertices and edges of a cube,
b) the complete bipartite graph $K_{m,n}$, where $m \geq n$,
c) the graph in Exercise 1.

5. The vertices of an 18-vertex graph G can be divided into 3 classes of six vertices each, in such a way that 2 vertices are adjacent if and only if they are in different classes. Determine the largest integer k for which G is k-vertex-connected ($\kappa(G)$), and the largest integer l for which G is l-edge-connected ($\lambda(G)$).

6. Show that a k-(vertex-)connected graph G on n vertices has at least $kn/2$ edges.

7. Prove that an $n/2$-(vertex-)connected graph on n vertices contains a Hamilton cycle.

8. Construct a simple graph which is 2-vertex-connected, 3-edge-connected and has minimum degree 4.

9. (MT'14) We connect two disjoint complete graphs on 5 vertices with 3 edges, in such a way that the resulting graph G is simple. Is it true in all cases that G is
a) 3-(vertex)-connected;
b) 3-edge-connected?

10. (MT'17) A simple graph on 10 vertices has 40 edges. Determine the largest integer k for which G is surely k-vertex-connected.

11. Show that if a graph is 3-(vertex-)connected, then it contains a cycle of even length.

12. (MT'07) Let G be a 3-(vertex-)connected graph with 100 vertices and let $x, y \in V(G)$ be two different vertices. Show that there is a path from x to y whose length (i.e. the number of edges in it) is not greater than 33.