

Introduction to the Theory of Computing 1.

Second Retake of the First Midterm Test

December 18, 2023

1. The number formed by the last two digits of 84 times the positive integer n is 1 greater than the number formed by the last two digits of n . Determine the last two digits of n .
2. The first term of an arithmetic progression is 600, and its difference is 77. (So the first few terms of the progression are 600, 677, 754, ...) Determine the remainder we get if we divide the product of the first 601 terms of the progression by 77.
3. Let $V \subseteq \mathbf{R}^5$ consist of those vectors \underline{v} for which at least one of the two conditions below hold:
 - 1) the first three coordinates of \underline{v} form an arithmetic progression (from top down);
 - 2) the last three coordinates of \underline{v} form an arithmetic progression (from top down).(E.g. $\underline{v} = (0, 8, 1, 5, 9)^T$ and $\underline{w} = (3, 3, 3, 3, 3)^T$ are in V .) Decide whether V forms a subspace in \mathbf{R}^5 or not.
4. What kind of geometric shape is the subspace spanned by the vectors $\underline{a}, \underline{b}, \underline{c}, \underline{d}$ in \mathbf{R}^3 below? If the subspace is a line or a plane, determine its (system of) equation(s).

$$\underline{a} = (2, 1, -1)^T, \underline{b} = (-6, -3, 3)^T, \underline{c} = (4, -1, -9)^T, \underline{d} = (8, 7, 3)^T$$

5. Let $V \subseteq \mathbf{R}^5$ be the set of vectors whose first four coordinates form an arithmetic progression and the last coordinate is the sum of the first 4 coordinates. E.g. $\underline{v} = (1, 3, 5, 7, 16)^T$ is in V . Determine a basis in V containing the vector \underline{v} above. (For the solution you don't need to show that V is in fact a subspace.)
6. * Determine the center of the circumscribed circle of the triangle with vertices $A = (79, 2, 37)^T$, $B = (-19, 72, 23)^T$ and $C = (59, 42, 47)^T$.

Please work on stapled sheets only, and submit all of them at the end of the midterm, including drafts. Write your name on every sheet you work on, and write your Neptun code on the first page.

You have 90 minutes to work on the problems. Each of them is worth 10 points. The problem marked with an * is supposed to be more difficult.

The details of the solutions must be explained, giving the result only is not worth any points. Notes, calculators (or similar devices) cannot be used.

Please turn over for the second midterm.