Introduction to the Theory of Computing 1.
First Retake of the First Midterm Test

November 17, 2023

1. On the third planet of the Great Vogon System, every month is 29 days long since ancient times, and the remaining 22 days at the end of the year are spent by the residents' excessive drinking of the famous pálinka. However, an in-depth analysis by the National Bank recently showed that the the 22-day break at the end of the year has an unfavorable effect on the performance of the economy. That is why the 32-day month is introduced, so the number of days omitted from months is reduced to 5. How many days does the year consist of on the third planet, if we know that the year on the fourth planet (which of course is longer than that of the third) is 1000 days long.

2. Determine the remainder we get if we divide 499^{4201} by 539.

3. Let $\underline{d} = (3, 0, -1)$ be the direction vector of both the lines e and f. Line e contains the point $(3, 1, 2)$, and line f contains the point $(5, -1, 1)$. Determine the equation of the plane containing both the lines e and f.

4. Let $V \subseteq \mathbf{R}^5$ be the set of vectors whose first four coordinates form an arithmetic progression and the last coordinate is the sum of the first 4 coordinates. (E.g. $\underline{v} = (1, 3, 5, 7, 16)^T$ is in V.) Decide whether V forms a subspace in $\mathbf{R}^5$ or not.

5. Determine those values of the parameter p for which the vectors $a \mid b$ $\underline{a} = (1, 1, 1, 1)^T$, $\underline{b} = (1, 1, 1, p)^T$ and $\underline{c} = (1, 1, p, p)^T$ can be extended to a basis in $\mathbf{R}^4$.

6. * Determine all the integers x for which $x^6 \equiv 2 \pmod{201}$ holds.

Please work on stapled sheets only, and submit all of them at the end of the midterm, including drafts.

Write your name on every sheet you work on, and write your Neptun code and the number of the group you are registered to in Neptun (A1, A2 or A3) on the first page.

You have 90 minutes to work on the problems. Each of them is worth 10 points. To obtain a signature you have to achieve at least 24 points on each of the two midterm tests.

The details of the solutions must be explained; giving the result only is not worth any points. Notes, calculators or any additional devices cannot be used. The problem marked with an * is supposed to be more difficult.