

Introduction to the Theory of Computing 1.

First Midterm Test

November 3, 2023

1. Determine all the integers n between 1 and 2023 for which both the fraction $\frac{n-2}{21}$ and the fraction $\frac{n-5}{166}$ has an integer value.
2. What is the remainder we get if we divide $6 \cdot 10^{23}$ by 9998?
3. A regular rectangular chest stands on a flat, sloping ground. The corner (vertex) of the chest $A(1, 4, 2)$ is on the ground, but the vertex $B(4, 2, 1)$ which is adjacent to A is not on the ground. Does the plane of the ground intersect the z axis? If yes, where?
4. Determine all those values of the parameter p for which the vectors $\underline{a} = (1, 1, 17, 23)^T$, $\underline{b} = (4, 5, 19, 29)^T$, $\underline{c} = (0, 0, 2, 5)^T$ and $\underline{d} = (0, 0, 4, p)^T$ are linearly independent.
5. The subspace V of \mathbf{R}^5 consists of those column vectors $\underline{x} \in \mathbf{R}^5$ whose first three coordinates form an arithmetic progression (from top down) and last three coordinates form a geometric progression with quotient 2 (also from top down). (E.g. $(1, 6, 11, 22, 44)^T$ is such a vector.) Determine the dimension of the subspace V . (For the solution you don't need to show that V is in fact a subspace.)
6. * Determine all the Fermat liars for $m = 1024$, that is, all the positive integers less than 1024 which in the Fermat test don't show that 1024 is composite.

Please work on stapled sheets only, and submit all of them at the end of the midterm, including drafts.

Write your name on every sheet you work on, and write your Neptun code and the number of the group you are registered to in Neptun (A1, A2 or A3) on the first page.

You have 90 minutes to work on the problems. Each of them is worth 10 points. To obtain a signature you have to achieve at least 24 points on each of the two midterm tests.

The details of the solutions must be explained; giving the result only is not worth any points. Notes, calculators or any additional tools cannot be used. The problem marked with an * is supposed to be more difficult.