Introduction to the Theory of Computing 1.
Second Repeat of the Second Midterm Test

2020. January 3.

1. Let's call a vector $\underline{v}$ in $\mathbf{R}^n$ a *palindrome*, if writing the coordinates of $\underline{v}$ in the reverse order we get $\underline{v}$ as well. (E.g. $(7,3,8,3,7)^T$ is a palindrome.) Determine the dimension of the subspace V containing all the palindromes in $\mathbf{R}^5$. (For the solution you don't need to show that V is in fact a subspace.)

2. Determine for which values of the parameters p and q the system of equations below is consistent. If it has solutions, then determine the number of them (but you don't need to determine the solutions themselves).
$$\begin{aligned} x_1+2x_2+5x_3+9x_4 &= 7\\ 2x_1+3x_2+8x_3+8x_4 &= 10\\ 3x_1+6x_2+8x_3+5x_4 &= 30\\ 3x_1+5x_2+13x_3+p\cdot x_4 &= q \end{aligned}$$

3. Evaluate the determinant below for all the values of the parameters a, b.
$$\begin{vmatrix} a & 2a & 4a & 9a\\ b & 2b & 5b & 10b\\ 1 & 3 & 6 & 12\\ 3 & 6 & 12 & 20 \end{vmatrix}$$

4. Let A be the matrix below. Does there exist a matrix X for which $A\cdot X$ is the 3×3 identity matrix?
$$A=\begin{pmatrix} 1 & 3\\ 3 & 8\\ 7 & 6 \end{pmatrix}$$

5. Determine the rank of the matrix below for all the values of $a, b, c \in \mathbf{R}$.
$$\begin{pmatrix} a & 2b & 3c\\ 1 & 2 & 3\\ a+1 & 2b+2 & 3c+3 \end{pmatrix}$$

6. * Show that there exists a 5×5 invertible matrix A which has exactly ten invertible 2×2 submatrices.

The full solution of each problem is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit.
Calculators (or other devices) are not allowed to use.