Introduction to the Theory of Computing 1.
Repeated Second Midterm Test
2019. December 16.

1. Let the subspace W of $\mathbf{R}^4$ consist of those column vectors whose second coordinate is twice as large, and third coordinate is three times as large as the first one. (E.g. the vector $(5, 10, 15, 39)^T$ is in W.) Determine the dimension of the subspace W. (For the solution you don't need to show that W is in fact a subspace.)

2. Let A be a 5×10 matrix whose rows are linearly dependent, and $\underline{b} \in \mathbf{R}^5$ a column vector. Show that if the system of linear equations $(A|\underline{b})$ is consistent, then it has a solution in which the values of at least 4 variables are different from 0.

3. Evaluate the determinant below (by any method).

$$\begin{vmatrix} 3 & 15 & 9 & 3 \\ -2 & -10 & 1 & -6 \\ 1 & 5 & 7 & -3 \\ 2 & 13 & 12 & -10 \end{vmatrix}$$

4. The 2×3 matrix A doesn't have negative entries. Furthermore, we know that the upper left entry of the matrix $A \cdot A^T$ is 0 and the lower right entry of it is 14, moreover the upper left entry of the matrix $A^T \cdot A$ is 4 and the lower right entry of it is 9. Determine the matrices A, $A \cdot A^T$ and $A^T \cdot A$.

5. Decide whether the matrix A below is invertible or not, and if yes, determine its inverse and the rank of its inverse.

$$A = \begin{pmatrix} 1 & -1 & 0 \\ -13 & 15 & -7 \\ 7 & -8 & 4 \end{pmatrix}$$

6. * Let U and V be such 10 dimensional subspaces in $\mathbf{R}^{20}$ whose common element is only the zero vector. Show that for each vector $\underline{x} \in \mathbf{R}^{20}$ there are vectors $\underline{u} \in U$ and $\underline{v} \in V$ for which $\underline{x} = \underline{u} + \underline{v}$.

The full solution of each problem is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit.

Notes and calculators (or other devices) are not allowed to use.