Introduction to the Theory of Computing 1.
Second Repeat of the First Midterm Test

2020. January 3.

1. Determine the remainder we get if we divide 7^{3234} by 80.
2. In the old British monetary system 1 crown was worth 60 pennies. In a company a boss had less than 50 subordinates in his group. The subordinates were earning 26 pennies each, while the boss' salary was 2 crowns. The boss got all the wages in his group in crowns, and after paying everybody, 2 pennies remained. How many subordinates did the boss have in his group?
3. The system of equations of the line e is $x = y = \frac{z-3}{2}$, and that of the line f is $\frac{x-2}{2} = \frac{y+1}{3} = z + p$, where $p \in \mathbf{R}$. Determine whose values of p for which e and f intersect.
4. Let's call a vector $\underline{v}$ in $\mathbf{R}^n$ a *palindrome*, if writing the coordinates of $\underline{v}$ in the reverse order we get $\underline{v}$ as well. (E.g. $(5, 1, 2, 1, 5)^T$ is a palindrome.) Show that the set of all the palindromes in $\mathbf{R}^5$ is a subspace of $\mathbf{R}^5$.
5. Let the vectors $\underline{a}, \underline{b}, \underline{c} \in \mathbf{R}^n$ be linearly independent. Does it follow that the vectors $2\underline{a}+\underline{b}$, $2\underline{b}+\underline{c}$ and $2\underline{c}+\underline{a}$ are also linearly independent?
6. * Determine whether the following statement is true or not:
   If $x^2 \equiv 1 \pmod{187}$ then $x \equiv 1 \pmod{187}$ or $x \equiv -1 \pmod{187}$.
   (If it is true, then prove it, if not, then give a counterexample.)

The full solution of each problem is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit.
Notes and calculators (and similar devices) cannot be used.