Introduction to the Theory of Computing 1.
First Midterm Test

2019. October 25.

1. The last two digits of the positive integer n in both of the numerical system of base 4 and in the numerical system of base 5 are 13 as well. What are the last two digits of n in the decimal system?

2. Show that $1010^{1343} - 2$ is divisible by 2019. (The prime factorisation of 2019 is: $2019 = 3 \cdot 673$.)

3. Use the algorithm we learnt to determine the remainder we get if we divide 10^{40} by 46.

4. * How many integers b exist between 2 and 2019 for which there exists two neighboring positive integers (n and $n+1$) in such a way that the sum of digits of both of them in the numerical system of base b is divisible by 2019?

5. The line e with system of equations $\frac{x-11}{2} = \frac{z+19}{-5},\ y = -1$ intersects the plane of equation $2x+y-2z = 3$ in the same point as the line f passing through the point $P(15, 2, -8)$. Determine the system of equations of f.

6. Let $\underline{u}, \underline{v}$ and $\underline{w}$ be the vectors in $\mathbf{R}^4$ below. Determine all those elements of the generated subspace $\langle \underline{u}, \underline{v}, \underline{w} \rangle$ whose all the coordinates are equal.

$$\underline{u} = (1, 2, 0, 0)^T,\ \underline{v} = (1, 0, 3, 0)^T,\ \underline{w} = (1, 0, 0, 4)^T$$

The full solution of each problem is worth 10 points. Show all your work!
Results without proper justificati on or work shown deserve no credit.
Notes and calculators (and similar devices) cannot be used.
The question denoted by an * is supposed to be more difficult.

Grading: 0-23 points: 1, 24-32 points: 2, 33-41 points: 3, 42-50 points: 4, 50-60 points: 5.