Introduction to the Theory of Computing 1. Repeated First Midterm Test

2018. December 10.

- 1. The last three digits of 513 times the integer n are 001. What are the last three digits on n?
- 2. Determine the remainder we get if we divide $169^{181^{194}}$ by 392.
- 3. Let n = 20181210. Use the algorithm we learnt to determine the g.c.d. of 45n + 12 and 35n + 9.
- 4. * Consider the triangle with vertices A(1,5,2), B(2,7,4) and C(2,9,10). Determine the (inner) bisector of the angle at vertex A. (The bisector of the angle at A is the line through A in the plane of the triangle for which the angles enclosed by it and the sides AB and AC respectively are equal.)
- 5. Let the set V consist of the vectors in \mathbb{R}^4 for which it holds that the product of their four coordinates is greater than or equal to 0. Decide whether V forms a subspace in \mathbb{R}^4 or not.
- 6. Let the set W consist of the vectors in \mathbb{R}^6 for which it holds that the odd indexed coordinates of them form a geometric sequence with quotient 2 (from top down), and the even indexed coordinates of them form a geometric sequence with quotient 3 (from top down). (E.g. the vector $(3, 5, 6, 15, 12, 45)^T$ is in W.) Determine the dimension of the subspace W. (For the solution you don't need to show that W is in fact a subspace.)

The full solution of each problem is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit.

Notes and calculators (or other devices) are not allowed to use. The question denoted by an * is supposed to be more difficult.