

Introduction to Computer Science I.
Second Repeat of the First Midterm Test
2017. December 18.

1. How many integers x are there between 1 and 2017 for which it holds that $92x - 1$ and x give the same remainder when divided by 399?
2. Let p be a positive prime number different from 3 and a be an integer not divisible neither by 3 nor by p . Show that in this case

$$a^{6p-6} \equiv 1 \pmod{9p}$$

3. Let $n = 123456$. Use the algorithm we learnt to determine the g.c.d. of $12n + 6$ and $9n + 4$.
4. The system of equations of the line e is $\frac{x+3}{5} = \frac{y+1}{9} = z$, and of the line f is $\frac{x}{4} = \frac{y+8}{6}, z = 7$. Determine the system of equations of the *normal transversal* of e and f , that is, of the line n which intersects both e and f perpendicularly.
5. Let $\underline{a} = (1, 2, 4)^T$, $\underline{b} = (0, 1, 2)^T$ and $\underline{c} = (0, 0, 1)^T$ be vectors in \mathbf{R}^3 .
 - a) Do the vectors $\underline{a}, \underline{b}, \underline{c}$ form a generating system in \mathbf{R}^3 ?
 - b) Do the vectors $\underline{a}, 3\underline{a} + \underline{b}, 6\underline{a} + 2\underline{b} + \underline{c}$ form a generating system in \mathbf{R}^3 ?
6. Determine whether the vectors $\underline{u} = (4, 3, 8, 1)^T$, $\underline{v} = (2, 0, 4, 0)^T$ and $\underline{w} = (3, 5, 6, 2)^T$ in \mathbf{R}^4 are linearly independent or not.

The full solution of each problem is worth 10 points. Show all your work!
Results without proper justification or work shown deserve no credit.
Notes and calculators (and similar devices) cannot be used.