Introduction to Computer Science I.
Second Repeat of the First Midterm Test

2015. December 18.

1. Determine the subspace spanned by the following sets of vectors in $\mathbf{R}^3$. If the subspace is a line or plane, then determine its (system of) equation(s).
   a) $(2, -6, 8)^T, (3, -9, 12)^T$,
   b) $(2, -6, 8)^T, (3, -9, 11)^T$.

2. Let $\underline{a}, \underline{b}$, and $\underline{c}$ be vectors in $\mathbf{R}^4$. Suppose that for any *integers* k, l and m not all of whose are 0, the linear combination $k \cdot \underline{a} + l \cdot \underline{b} + m \cdot \underline{c}$ is not the zero vector. Does it follow that $\underline{a}, \underline{b}, \underline{c}$ is a linearly independent set?

3. Let the set V consist of those vectors in $\mathbf{R}^5$ for which it holds that if we add an appropriate common number to each of their coordinates then we get a vector whose coordinates form a geometric progression with quotient 2. (E.g. the vector $(2, 7, 17, 37, 77)^T$ is like that, because if we add 3 to each of its coordinates we get a geometric progression with quotient 2.) Decide whether V forms a subspace in $\mathbf{R}^5$ or not. If yes, then determine the dimension of V.

4. No matter how we omit one equation from a system of linear equations with four variables the system obtained will have a unique solution. At least how many equations should the original system of linear equations contain?

5. Evaluate the determinant below (by any method).

$$\begin{vmatrix} 1 & -3 & 2 & 3 & 0 \\ 2 & -6 & 4 & 11 & -7 \\ -3 & 9 & -5 & -9 & 5 \\ 4 & -12 & 8 & 13 & -2 \\ 5 & -11 & 10 & -1 & -12 \end{vmatrix}$$

6. Compute the matrix A^{101} for the matrix A below.

$$A = \begin{pmatrix} 5 & -3 & 0 \\ 8 & -5 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

The full solution of each problem is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit. Calculators (or other devices) are not allowed to use.