List of Questions

1. Divisibility, prime numbers, fundamental theorem of arithmetic**, $d(n)$ function*, greatest common divisor, least common multiple. Number of primes**, gap between adjacent primes**, prime number theorem (no proof).

2. Congruences, operations with congruences**. Linear congruences, their solvability** and methods for their solutions. Simultaneous congruence systems (example).

3. Euler's $\varphi(n)$ function*, reduced residue system. Euler-Fermat theorem**, little Fermat theorem**. Euclidean algorithm*, its application for solving linear congruences (example).

4. Polynomial algorithms. Number theoretic algorithms: basic operations, exponentiation. Primality testing*, public key criptography, RSA-encoding*.

5. Geometry of 3-space: equations of planes**, lines**; intersections. Dot product, cross product: definitions, properties, evaluation.

6. Definition of $\mathbf{R}^n$ and subspaces of $\mathbf{R}^n$. Linear combination, spanned (generated) subspace, generating system, linear independence (2 definitions and their equivalence**), exchange theorem**, I-G inequality**.

7. Basis, dimension**. Standard basis, the dimension of $\mathbf{R}^n$*. Coordinate vector in a basis, its uniqueness**. Existence of a basis in a subspace of $\mathbf{R}^n$**.

8. Systems of linear equations, Gaussian elimination. Row echelon form, reduced row echelon form. Conditions on consistency (solvability) and uniqueness**.

9. Determinant: definition, basic properties**, ways of evaluation, expansion theorem*.

10. Matrices, operations on matrices, their properties. Product theorem for determinants (no proof). Connections between systems of linear equations and matrix equations**.

11. Inverse of a matrix, necessary and sufficient condition for its existence**, calculation of the inverse*. Rank of a matrix: definitions, evaluation (no proof).

12. Linear maps: definition, basic properties**, examples. Matrix of a linear map**. Composition (product) of linear maps, its matrix*. Inverse of a linear transformation*.

13. Kernel and image of linear maps**, examples. Dimension theorem*. Changing bases, the matrix of a linear transformation in a given basis**.

14. Eigenvalues and eigenvectors of linear transformations and of matrices, their determination**, characteristic polynomial, examples. Diagonalisation**.

Theorems and statements with an * were partially proved in the lecture.
Theorems and statements with a ** were completely proved in the lecture.