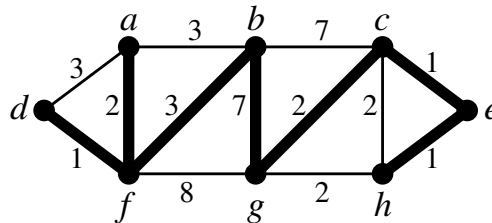


Second Retake of the Midterm Test

1. The simple graph G on 10 vertices has an edge which is contained in every cycle of G .
 - a) Is it possible that the number of edges of G is 10?
 - b) Is it possible that the number of edges of G is 11?
2. Decide whether the following statement is true or false: If G is any simple, connected graph not containing an Euler circuit then we can add a new vertex to it and connect it to some vertices of G (with one edge) in such a way that the graph obtained contains an Euler circuit.
3. Decide whether the thickened edges form a minimum weight spanning tree in the graph below or not.



4. The vertex set of the simple graph G is $V(G) = \{10, 11, \dots, 49\}$. Two different vertices are adjacent in G if and only if the corresponding numbers don't have a common digit. (E.g. 17 is adjacent to 35 and 42 in G , but it is not adjacent to 13, 27 or 41.) Determine $\chi(G)$, the chromatic number of G .
5. The vertex set of the simple graph G is $V(G) = \{1, 2, 3, \dots, 100\}$, and two vertices are adjacent in G if and only if the larger of the two numbers is at least twice as large as the smaller one. Determine a maximum independent set (of vertices) and a minimum edge cover in G .
6. * In the network (G, s, t, c) the value of the maximum flow from s to t is 10. If we increase the capacity of the edge e by 1 and decrease the capacity of the edge f by 1 then the value of the maximum flow remains 10. Then we decrease the capacity of the edge e by 1 and increase the capacity of the edge f by 1 (from the original values). Is it possible then that the value of the maximum flow is
 - a) 9,
 - b) 10,
 - c) 11?
 (We can suppose that in the original network the capacities of both e and f were at least 1.)

Please work on stapled sheets only, including drafts, and submit all of them at the end of the midterm.

Write your name on every sheet you work on, and write your Neptun code on the first page.

The details of the solutions must be explained; giving the result only is not worth any points. Notes, calculators or any additional devices cannot be used.

You have 90 minutes to work on the problems. Each of them is worth 10 points. The problem marked with an * is supposed to be more difficult. To obtain a signature you have to achieve at least 24 points.