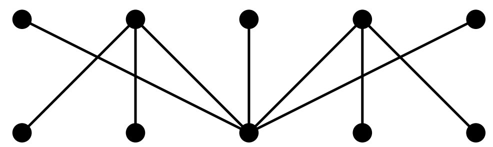
1. For the simple graph G on 10 vertices the following holds: no matter how we list k different vertices $v_1, v_2, \dots, v_k$ for $3 \le k \le 10$, at least one of the pairs $\{v_1, v_2\}, \{v_2, v_3\}, \dots,$ $\{v_{k-1}, v_k\}$ and $\{v_k, v_1\}$ is in the edge set of G. Show that G has at least 36 edges.

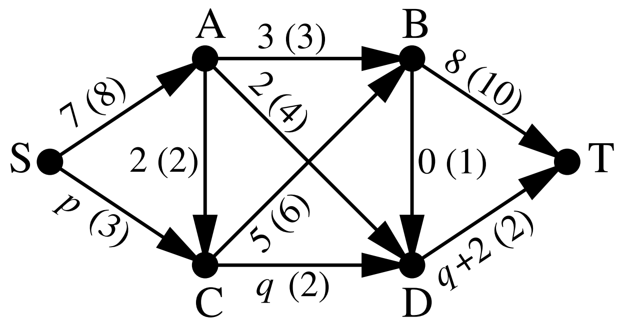
2. The vertex set of the simple graph G is $V(G) = \{1, 2, 3, \dots, 8\}$. In G, 1 is adjacent to 8, and beside this the vertices $x, y \in V(G),\ x \ne y$ are adjacent in G if and only if $|x - y| \le 2$. What is the length of the longest trail in G? (The length of a trail is the number of its edges.)

3. At least how many edges have to be added to the graph below so that the graph obtained has a perfect matching?



4. The vertices of the graph G are the 0-1 sequences of length four, and two vertices are adjacent if the corresponding sequences differ in exactly one digit. Determine $\chi_e(G)$, the edge-chromatic number of the graph G.

5. a) For which values of the parameters p and q will the values on the edges of the graph below form a flow from S to T?
   b) Is it true that for these values of p and q we get a maximum flow in the network?



6. The chromatic number of the simple graph G is $\chi(G) = 10$, and the minimum size of a vertex cover in G is $\tau(G) = 9$. Determine $\omega(G)$, the size of a maximum clique in G.

Please work on stapled sheets only, including drafts, and submit all of them at the end of the midterm.
Write your name on every sheet you work on, and write your Neptun code and the name of your practice instructor (according to neptun) on the first page.

You have 90 minutes to work on the problems. Each of them is worth 10 points. The problem marked with an * is supposed to be more difficult. To obtain a signature you have to achieve at least 24 points.

The details of the solutions must be explained; giving the result only is not worth any points. Notes, calculators or any additional tools cannot be used.