Second Retake of the Midterm Test

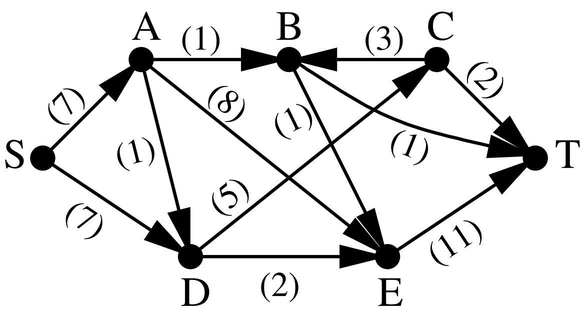
1. The vertex set of the simple graph G is $V(G) = \{1, 2, 3, \dots, 8\}$. In G, 1 is adjacent to 8, and beside this the vertices $x, y \in V(G)$, $x \neq y$ are adjacent in G if and only if $|x - y| \leq 2$. Is G isomorphic to its own complement?

2. Let the vertices of the graph G be those points of the plane whose coordinates are both integers between 1 and 4 (including 1 and 4 as well). (With a formula: $V(G) = \{(x, y) \in \mathbf{R}^2 : 1 \leq x, y \leq 4, x, y \in \mathbf{Z}\}$.) Let the different $P = (a, b)$ and the $Q = (c, d)$ vertices be adjacent in G if $|a - c| \leq 1$ or $|b - d| \leq 1$, that is, $P \neq Q$ differ by at most 1 in one of their coordinates. Determine $\chi(G)$, the chromatic number of G.

3. We construct the graph G on 8 vertices by adding 3 isolated vertices to a complete graph on 5 vertices then taking the complement of the graph obtained. Determine $\rho(G)$, the minimum number of covering edges, and give a minimum edge cover in G.

4. Somebody ran the BFS algorithm (its directed graph version) on the graph below, and wrote down the order in which the algorithm visited the vertices. However, four letters in the sequence became illegible, these are replaced by squares. Decide whether the following can be the remaining incomplete sequences or not. If the answer is yes, complete the sequence with the missing vertices and determine the corresponding BFS tree. (The numbers on the edges are not important here.)
   a) $S, \square, D, C, \square, \square, \square$ b) $S, \square, D, E, \square, \square, \square$.

5. a) Determine the capacity of the cut with $X = \{S, D\}$ in the network to the right.
   b) Is it true that the cut with $X = \{S, D\}$ is a minimum cut in this network?



6. * Let G be a connected graph and $w : E(G) \to \mathbf{R}$ be a weight function on the edges of G. Moreover, let $e \in E(G)$ be an edge of G and let $Z = \{f \in E(G) : w(f) \leq w(e)\}$ (i.e. Z consists of the edges of G whose weight is not larger than $w(e)$). Show that if $Z \setminus \{e\}$ doesn't contain the edges of any spanning tree of G, but Z does, then all minimum weight spanning trees of G contain e.

Please work on stapled sheets only, including drafts, and submit all of them at the end of the midterm.
Write your name on every sheet you work on, and write your Neptun code on the first page.

You have 90 minutes to work on the problems. Each of them is worth 10 points. The problem marked with an * is supposed to be more difficult. To obtain a signature you have to achieve at least 24 points.

The details of the solutions must be explained; giving the result only is not worth any points. Notes, calculators or any additional tools cannot be used.