Repeat Midterm Test

1. Let the vertices of the graph G be all the 5-element subsets of a 12-element set, and two vertices be adjacent if and only if the corresponding subsets are disjoint. How many edges does G have?

2. Let G be the complete graph on the vertex set $V(G) = \{1, 2, \ldots, 10\}$. For every $1 \leq i < j \leq 10$ let the weight of the edge $\{i, j\}$ be $\lfloor \frac{2j-i}{3} \rfloor$ (where $\lfloor\ \rfloor$ denotes the lower integer part). Determine a minimum weight spanning tree in G.

3. How many non-isomorphic complete bipartite graphs are there on 100 vertices which contain a Hamilton cycle? (In a complete bipartite graph all the verttices in one class are connected to all the vertices in the other class.)

4. Let the two vertex classes of the bipartite graph $G(A, B; E)$ be $A = \{a_1, a_2, \ldots, a_7\}$ and $B = \{b_1, b_2, \ldots, b_7\}$. For each $1 \leq i \leq 7$ and $1 \leq j \leq 7$ let a_i and b_j be adjacent if the entry in the ith row and jth column of the matrix below is 1. Determine a maximum matching and a minimum covering set in G.

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

5. Let the graph G be the complement of a path on 6 vertices (with 5 edges). Determine $\chi_e(G)$, the edge-chromatic number of the graph G.

6. * Let the network (G, s, t, c) be given, furthermore an edge e in G for which $c(e) > 0$ holds. Determine whether the statements below are true or not:
   a) If there is a minimum s, t-cut C for which e goes out of X, then $f(e) = c(e)$ holds for all maximum flows f.
   b) If $f(e) = c(e)$ holds for all maximum flows f, then there is a minimum s, t-cut C for which e goes out of X.

Total work time: 90 min + 20 minutes for uploading. Late turn-ins are not accepted.

The full solution of each problem (including explanations) is worth 10 points. Show all your

work! Results without proper justification or work shown deserve no credit.

Cooperation with each other is strictly forbidden!