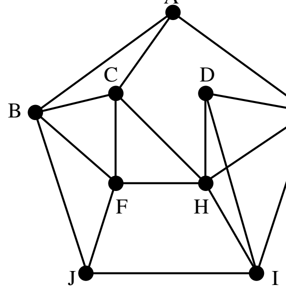
ITC2+ CGT1 **Second Repeat Midterm Test Grading Guide**

General Rules. Disclaimer: Google translate has been used for this section. The purpose of the scoring guide is to ensure that the dissertations are evaluated uni- formly by the correctors. Therefore, the guide the main ideas for solving each task (at least one possible) and the marks assigned to them communicates sub-scores. The guide is not intended to detail the complete solution of the tasks description; the steps described can be considered as a sketch of a solution with a maximum score. The sub-scores indicated in the guide only accrue to the solver if the related idea is included in the dissertation as a step towards a clear, clearly described and justified solution. Thus, for example, stating the definitions and items in the material without knowing how to apply them does not deserve any points (even if any of the facts described are indeed used in the solution). Deciding the score based on the points indicated in the guide in light of the above is under the grader's full remedial authority. A partial score is awarded for each idea or partial solution from which, with a suitable addition, a flawless solution to the problem would have been obtained. If a solver starts several several substantially different solutions for a task, he can be assigned to at most one score. If all the solutions or parts of solutions described are correct or correct, then the solution initiative worth the most subpoints is evaluated. However, if amongst several solution attempts there is a correct solution but also an incorrect one (with a substantial error), and it is not clear from the dissertation which the solver considered as correct, then the solution with fewer points is evaluated (even if this score is 0). The sub-scores in the guide can be further divided if necessary. A good solution other than that described in the guide is, of course, worth a maximum point. Theorems can be stated without proof, but only those discussed in class.

1. The local government of a small town has 20 members. They want to select a 5-member committee from themselves for organizing festivities. Some of the members get a raised reward from the mayor's budget. This will be decided by the mayor, who can give the raised reward to any number of the committee members (from 0 to 5). How many committees can be formed this way? (Two committees are considered different not only if their members are not the same but also if the members getting the raised reward are different.)

   Solution:
   We can choose the members in $\binom{20}{5}$ ways. (2 points)
   For a given committe we can decide about the raised rewards in 2^5 ways, (3 points)
   because .... (2 points)
   So the final anwswer is $\binom{20}{5} \cdot 2^5$, (2 points)
   because.... (1 point)

2. For which edges e of the graph G below (left) does it hold that there is a trail in G which contains all the edges of G except e? Determine such a trail for all such edges e as well (by the order in which it visits the vertices of G).



Solution:
The question is an Euler trail in $G - e$. (2 points)
(Necessary) condition: 0 or 2 odd-degree vertices. (1 point)
G has 4 odd-degree vertices, we have to select an edge with endpoints from them.
. (1 point)
The only possibility is to delete $e = \{D, H\}$, because..... (1+1 points)
Then there is an Euler trail in $G - e$, namely.... (4 points)

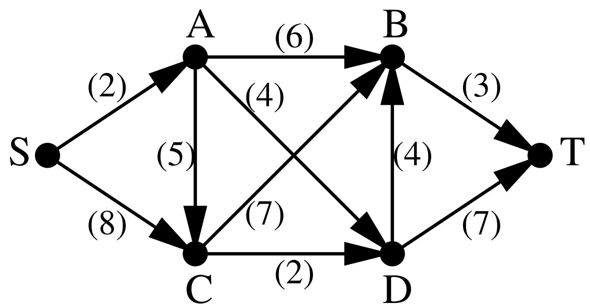
3. Determine the chromatic number of the graph G above (left).

Solution:
The vertices D, E, H, I form a clique, (2 points)
so $\chi(G) \geq \omega(G) \geq 4$. (2 points)
There is a good coloring with 4 colors, (3 points)
so $\chi(G) \leq 4$. (2 points)
Therefore $4 \leq \chi(G) \leq 4$, and $\chi(G) = 4$. (1 point)

4. Let G be a connected graph on 20 vertices. We know that no matter how we choose 8 edges of G there will be a vertex of G which is incident to at least 2 of them. Show that in this case no matter how we choose 12 edges of G there will be a vertex of G which is not incident to any of them.

Solution:
From the condition we have $\nu(G) \leq 7$, (2 points)
because otherwise..... (2 points)
What we need to prove is that $\rho(G) \geq 13$. (2 points)
because otherwise..... (2 points)
By Gallai's theorem (2): $\nu(G) + \rho(G) = 20(= |V(G)|)$ holds for G, (1 point)
so from $\nu(G) \leq 7$ we get $\rho(G) \geq 13$. (1 point)

5. Determine a maximum flow from S to T and a minimum S,T-cut in the network above (right).



Solution:
There is a flow of value 7. (4 points)

The capacity of the s,t-cut with $X = \{S, B, C\}$ is 7, (3 points)
because... (1 point)
We know that $7 \leq \max m(f) = \min c(C) \leq 7$, (1 point)
therefore the flow is maximum and the cut is minimum. (1 point)

6. * In a connected graph G the *distance* of the vertices u and v is the length (i.e. the number of edges) of a shortest path between u and v. The *diameter* of G is the maximum distance between two vertices in G (i.e. the distance of the farthest vertices). Show that if G is a simple graph on 24 vertices in which one vertex has degree 5 and all the other vertices have degree 3 then the diameter of G is at least 4.

   Solution:
   Let v be the vertex of degree 5. If we start a BFS from v, then there are 5 vertices of distance 1, and at most 10 vertices of distance 2 from v. (3 points)
   These are at most 16 vertices, so there must be vertices of distance at least 3 from v. Let s be one of them. (1 point)
   If we start a BFS from s, then there are 3 vertices of distance 1, at most 6 vertices of distance 2, and at most 12 vertices of distance 3 from s. (4 points)
   These are at most 22 vertices, so there must be vertices of distance at least 4 from s. So the diameter of G is at least 4. (1+1 points)