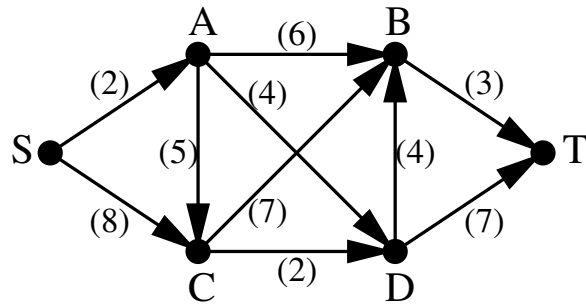
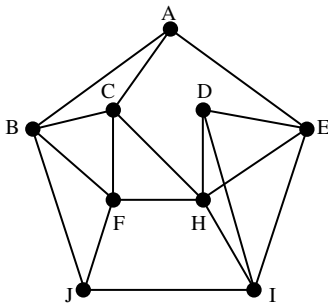


## Second Repeat Midterm Test

- The local government of a small town has 20 members. They want to select a 5-member committee from themselves for organizing festivities. Some of the members get a raised reward from the mayor's budget. This will be decided by the mayor, who can give the raised reward to any number of the committee members (from 0 to 5). How many committees can be formed this way? (Two committees are considered different not only if their members are not the same but also if the members getting the raised reward are different.)
- For which edges  $e$  of the graph  $G$  below (left) does it hold that there is a trail in  $G$  which contains all the edges of  $G$  except  $e$ ? Determine such a trail for all such edges  $e$  as well (by the order in which it visits the vertices of  $G$ ).



- Determine the chromatic number of the graph  $G$  above (left).
- Let  $G$  be a connected graph on 20 vertices. We know that no matter how we choose 8 edges of  $G$  there will be a vertex of  $G$  which is incident to at least 2 of them. Show that in this case no matter how we choose 12 edges of  $G$  there will be a vertex of  $G$  which is not incident to any of them.
- Determine a maximum flow from  $S$  to  $T$  and a minimum  $S, T$ -cut in the network above (right).
- \* In a connected graph  $G$  the *distance* of the vertices  $u$  and  $v$  is the length (i.e. the number of edges) of a shortest path between  $u$  and  $v$ . The *diameter* of  $G$  is the maximum distance between two vertices in  $G$  (i.e. the distance of the farthest vertices). Show that if  $G$  is a simple graph on 24 vertices in which one vertex has degree 5 and all the other vertices have degree 3 then the diameter of  $G$  is at least 4.

Total work time: 90 min + 20 minutes for uploading. Late turn-ins are not accepted. The full solution of each problem (including explanations) is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit.

**Cooperation with each other or an outer helper is strictly forbidden!**