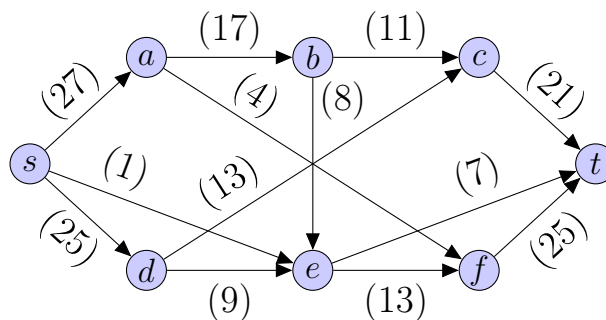


- Let  $G$  be a simple undirected graph, and let  $a$  and  $b$  be two different vertices of it. Furthermore, we know that in a BFS started from vertex  $a$  the fifth vertex found is  $b$ . Is it true then that there is a BFS started from vertex  $b$  in which the fifth vertex found is  $a$ ? (If the answer is yes, prove it, if no, give a counterexample.)
- (\*) A simple, connected graph on 20 vertices has 22 edges. Decide whether the graph is planar or not. (If the answer is yes, prove it, if no, give a counterexample.)
- In the simple graph  $G$  on 8 vertices there is no isolated vertex and the degree of each vertex is even. Show that we can add one edge to  $G$  in such a way that the graph obtained is still simple and contains an Euler trail.
- The simple graph  $G$  on 9 vertices consists of a cycle on 3 vertices and a cycle on 7 vertices with exactly one vertex in common. Determine the chromatic number of the complement of  $G$ .
- In a simple graph on 20 vertices the degree of each vertex is at least 10. Is it true then that the size of a minimum covering set of vertices is also at least 10?
- Determine a minimum cut in the network below.



Total work time: 90 min + 30 minutes for uploading. Late turn-ins are not accepted.

The full solution of each problem (including explanations) is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit.

Cooperation with each other is strictly forbidden!