# Test for the Signature

1. We want to make a new password for ourselves for safety reasons. We want the following conditions to be satisfied:
   a) it should consist of letters only, moreover, exactly 8 of them (from the 26 letters of the English alphabet),
   b) no letter should appear more than once,
   c) both upper and lowercase letters can appear, but one letter in only one form.
   How many passwords can we choose from with these conditions?

2. For a simple, connected planar graph G on 100 vertices it holds that we can delete the edges of one of its spanning trees in such a way that the remaining graph, H, is still connected. Show that if we delete the edges of a spanning tree from H, then the remaining graph has at least 4 components.

3. Determine a maximum matching in the graph below.
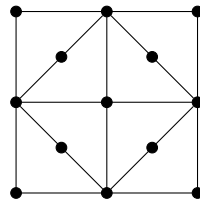
4. We delete the edges of a Hamilton cycle from K_8, the complete graph on 8 vertices. Determine the chromatic number of the graph obtained.

5. In a simple graph on 50 vertices the maximum degree is 7. Show that we can find 7 independent vertices in the graph.

6. (∗) Does there exist a simple graph whose edge-chromatic number is 5, but if we delete the edges of a Hamilton cycle from it then the edge-chromatic number of the graph obtained is only 2?

Total work time: 90 min + 30 minutes for uploading. Late turn-ins are not accepted. The full solution of each problem (including explanations) is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit. **Cooperation with each other or an outer helper is strictly forbidden!**